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PATHS OF TARGET-SEEKING MISSILES IN TWO DIMENSIONS

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SUMMARY

A rather detailed discussion is given of some of the parameters that enter into the equation of the path of a missile which is made to pursue a moving target. The missile was assumed to be guided by a target seeker that might be employed in any one of several ways. Four particular methods of employing a target seeker were assumed for discussion. These methods and the designations of the resulting motion are:

(1) If the target seeker is installed in the missile so that the axis of the field of view of the target seeker is always parallel to the longitudinal axis of the missile, the resulting motion is called normal pursuit navigation.

(2) If the target seeker is installed in the missile so that the axis of the field of view of the target seeker makes a constant angle with the longitudinal axis of the missile, the resulting motion is called constant navigation.

(3) If the target seeker is installed in the missile in such a way that the axis of its field of view turns through an angle that is a definite proportion of the angle turned through by the axis of the missile, the resulting motion is called proportional navigation.

(4) If the target seeker is installed on or near the launching station and made to direct a beam of radio signals at the target so that the missile can be made to fly in this beam of signals to the target, the resulting motion is called line-of-sight navigation.

For each method considered for use of a target seeker, the differential equations of highly simplified two-dimensional pursuit paths are presented. In the cases of normal pursuit and constant navigation, analytical solutions of the differential equations of the simplified pursuit paths were obtained and evaluated for some illustrative examples. It is shown that, for these cases, pursuit paths with finite curvature are obtained only if the ratio of the velocity of the missile to that of the target is less than 2. Differential-analyzer solutions of the equations for proportional navigation are presented for different rates of navigation and for different initial conditions. The pursuit paths resulting from proportional navigation generally improve as the rate of navigation is increased and/or as the ratio of the velocity of the missile to that of the target is increased. Proportional navigation is concluded to be by far the best of the methods considered for using a target seeker installed in a missile. The equation of line-of-sight paths was reduced to functions of elliptic integrals and to a simpler form for numerical evaluation. They were evaluated for some particular conditions. The curvature of line-of-sight paths is generally finite. In all cases the effect on the nature of the pursuit paths of the important factors of relative speeds of the missile and target and the relative locations and directions of flight of the missile and target at the beginning of the pursuit are brought out.

The effects of increased angle of attack and sideslip due to turning were not taken into account in the analysis but these effects are briefly discussed. The work is intended only as a guide to be used in determining the manner in which a target seeker can be used to best advantage for a given problem.

INTRODUCTION

It is proposed that a missile of the type that flies as an ordinary airplane can be made to pursue and intercept a moving target by equipping the missile with an automatic pilot which would be actuated by signals from a target seeker. A target seeker is a device that can detect the presence of certain objects or targets within its field of view by one of several methods such as by heat, light, or sound emitted from the target or by recently developed radio equipment.

The field of view of the target seeker can be considered as a cone with its vertex in the seeker. As a target moves in this cone, the target seeker can be arranged to give signals that are related to the displacement of the target from the axis of the cone and the distance of the target from the target seeker. These signals can thus be used to direct the automatic pilot to guide the missile toward the target.

It is further proposed that, for the purpose of guiding a missile to a target, the target seeker may be installed either in the missile or on or near the device used for launching the missile. If installed in the missile, the target seeker may be arranged either to have some specified fixed position or to turn in some prescribed manner and, in either case, to transmit signals directly to the automatic pilot. If installed on or near the launching device, the target seeker could be arranged to turn in all directions so that the target can be kept automatically in the center of its field of view and thus directly contacted by a beam of radio signals. By means of radio equipment the missile can then be made to fly in this radio beam and thereby to remain between the target seeker and target until it reaches the target. The missile may be launched from an airplane, a ship, or a stationary ground station.

If the assumption is made that a missile, equipped in some manner as previously proposed, will pursue a moving target and that the target-seeking equipment does not render it dynamically unstable, the nature of the theoretical flight path (which will hereinafter be called pursuit path) and thus the maneuvers (direction and rate of turning) required of it to follow the path depend on the path of the target, the relative speeds of the target and missile, the relative directions of flight and locations of the target and missile at the beginning of the pursuit, and finally, to a great extent, on the manner in which the target seeker is employed.

If a comparison of pursuit paths that might be obtained under different conditions is desired, some qualitative bases must be established by which the paths can be compared. Inasmuch as the adverse aerodynamic effects encountered in turning an aircraft increase with the rate of turning, the principal basis for comparison should be the rate of turning required of the missile to follow the path. The path that requires a minimum rate of turning therefore can generally be considered the best path. Other factors to be considered are the point along the path at which the maximum rate of turning is required, the accuracy with which certain required parameters leading to a particular type of path can be determined within the time that may be allotted, and the direction from which the missile approaches the target at the end of the pursuit.

For any particular method of employing a target seeker, the problem of determining the actual flight path is in general an exceedingly difficult task that involves an enormous amount of computation. Nevertheless, by choosing different methods of employing a target seeker, neglecting aerodynamic effects, and making some highly simplifying assumptions, theoretical paths can be obtained that indicate the effects that some of the different factors determining the flight path have on the nature of the path. Knowledge gained from the study of simplified cases serves as a guide in arriving at a satisfactory method of employing a target seeker and suitable values of any arbitrary parameters that might affect the nature of the flight paths for the more general and difficult cases.

The purpose of the present paper is to investigate some possible methods of using a target seeker and to derive corresponding pursuit paths. A comparison is made of pursuit paths obtained under similar conditions for different methods of using a target seeker. The effect on the nature of the paths of the important factors of relative speeds of the missile and target and the relative locations and directions of flight of the missile and target at the beginning of the pursuit are discussed in detail.

SYMBOLS

The symbols used in this discussion are defined as follows:

x, y Cartesian coordinates denoting position of missile at any given time

x_T, y_T coordinates of target at any given time referred to coordinate system of missile a distance from vertical axis to target path

TS axis of field of view of target seeker

t time

V velocity of missile

V_T velocity of target

N ratio of velocity of missile to velocity of target (V/V_T)

R distance from missile to target at any given time

γ angle (positive in counterclockwise direction) between direction of flight and reference line joining initial position of missile and target

ϵ angle (positive in counterclockwise direction) between axis of field of view of target seeker and reference line joining initial position of missile and target

Ω angle (positive in counterclockwise direction) between X-axis and reference line joining initial position of missile and target

$\left(\Omega \text{ is always in the range } -\frac{\pi}{2} < \Omega < \frac{\pi}{2}\right)$

σ constant angle (positive in counterclockwise direction) that longitudinal axis of missile makes with axis of target-seeker field of view; applies only to constant navigation

n ratio of angle turned through by missile to angle turned through by target seeker (γ/ϵ). Measured relative to reference axis; applies only to proportional navigation

β angle of sideslip

K rate of curvature $\left(\frac{d\gamma}{dt}\right)$

$$A = \frac{N \cos \sigma}{\sqrt{1 - N^2 \sin^2 \sigma}}$$ (applies only to constant navigation)

E arc sin (N sin σ) (applies only to constant navigation)

$$P = \frac{dy}{dx}$$

Subscripts:

O initial values

max maximum

SIMPLIFYING ASSUMPTIONS

Besides neglecting aerodynamic effects the following simplifying assumptions are made:

(1) The target moves in a straight line.

(2) The missile and target move at constant speeds and remain in the same geometrical plane.

(3) The target seeker, automatic pilot and missile respond immediately to correct any deviation of the target from the axis of the field of view of the target seeker.

(4) The missile always moves in the direction of its longitudinal axis.

Although the aerodynamic effects are totally neglected in deriving the equations of the pursuit paths discussed herein, the final section of the report presents a brief discussion of the effect that change in angle of attack and sideslip due to turning might have on the paths.

METHODS ASSUMED FOR USING A TARGET SEEKER

The different methods assumed for using a target seeker and the corresponding terms used to designate the method and the resulting pursuit curve are explained in the paragraphs that follow.

Normal-pursuit navigation.- If the target seeker is assumed to be fixed in the missile so that the axis of the field of view of the target seeker is parallel to the longitudinal axis of the missile, the missile would presumably always go directly toward the target. The motion is called normal-pursuit navigation and the pursuit path resulting from such motion is called a normal-pursuit path. Figure 1(a) shows the geometrical setup used to determine the equations of such a path. The longitudinal axis of the missile, the direction of its motion and the axis of the target-seeker field of view are all made to coincide. The differential equations of the normal-pursuit path can be integrated by simple methods and reduced to simple formulas for computation.

Constant navigation.- If the target seeker is assumed to be fixed in the missile with the axis of its field of view at a given angle relative to the longitudinal axis of the missile in such a way that the missile always leads the target, the resulting motion of the missile is called constant navigation. Figure 1(b) shows the geometrical setup used to determine the equations of the pursuit path resulting from constant navigation. The longitudinal axis of the missile and its direction of flight are assumed to coincide. The differential equations of the pursuit path resulting from constant navigation can be integrated by simple methods but the resulting formulas are tedious to handle in computational work.

Proportional navigation.- If the target seeker is installed in the missile in such a way that the axis of its field of view turns through an angle that is a definite proportion of the angle turned through by the longitudinal axis of the missile, the resulting motion of the missile is called proportional navigation. Figure 1(d) shows the geometrical setup used to determine the equations of the pursuit path resulting from proportional navigation. The longitudinal axis of the missile and the direction of its motion are assumed to coincide. The differential equations of the pursuit path resulting from proportional navigation cannot be integrated in closed form; however, they are easy to evaluate by numerical methods. Solutions were obtained for this report of a number of special cases on the differential analyzer of the General Electric Company.

<u>Line-of-sight navigation</u>.- If the target seeker is installed on or near the apparatus used for launching the missile and is made to direct a beam of radio signals to the target for the missile to fly in, the resulting motion is called line-of-sight navigation, and the resulting pursuit curve is called line-of-sight path. Figure 1(e) shows the geometrical setup for this case. Although the differential equation of the line-of-sight path cannot be integrated, it can be reduced to form suitable for numerical evaluation or it can be reduced to parametric functions of elliptic integrals of the first and second kinds.

DISCUSSION

The differential equations of pursuit paths resulting from the different types of navigation defined will be solved, when possible. The derivation of the differential equations of normal pursuit paths will be taken up in detail and, since the equations for constant and proportional navigation can be derived in an analagous manner, it will suffice when those cases are reached simply to write the appropriate equation.

Normal-Pursuit Navigation

<u>Equations of normal-pursuit path</u>.- In figure 1(a) the Cartesian coordinate axes are chosen so that the origin represents the initial position of the missile and (x, y) its position at any time t. The x-axis is chosen perpendicular to the path line of the target so that the equation of the path of the target becomes $x = a$. The target is assumed to move in a positive direction along this line from its initial position (a, y_{T_0}) to (a, y_T).

With the geometry of the problem thus established the required equation can be obtained. By equating equal expressions of time, the equation

$$\frac{\sqrt{(dy_T)^2}}{V_T} = \frac{\sqrt{(dx)^2 + (dy)^2}}{V}$$

or

$$\frac{V}{V_T}\left(\frac{dy_T}{dx}\right) = N\frac{dy_T}{dx}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \qquad (1)$$

is obtained. Since the target is always in the tangent to the path of the missile a second relation

$$y_T - y = \frac{dy}{dx}(a - x) \qquad (2)$$

is obtained.

Equations (1) and (2) can be combined and integrated in Cartesian form but it will be convenient for other considerations to make the following substitutions:

$$\left.\begin{aligned} a - x &= R \cos (\epsilon + \Omega) \\ y_T - y &= R \sin (\epsilon + \Omega) \\ y_T &= y_{T_0} + V_T t \end{aligned}\right\} \qquad (3)$$

With these substitutions the equations of the path become

$$\left.\begin{aligned} \frac{d\epsilon}{dt} &= \frac{V_T}{R} \cos (\epsilon + \Omega) \\ \frac{dR}{dt} &= V_T \sin (\epsilon + \Omega) - V \end{aligned}\right\} \qquad (4)$$

or

$$\frac{dR}{R} = [\tan(\epsilon + \Omega) - N \sec(\epsilon + \Omega)]\, d\epsilon \tag{5}$$

Integrating equation (5) and evaluating the constant of integration at $t = 0$ gives

$$R = R_0 \cos\Omega \left(\frac{1 + \sin\Omega}{1 - \sin\Omega}\right)^{N/2} \sec(\epsilon + \Omega) \left[\frac{1 - \sin(\epsilon + \Omega)}{1 + \sin(\epsilon + \Omega)}\right]^{N/2} \tag{6}$$

In equation (6), as the value of R ranges from R_0 to zero the value of ϵ ranges from zero to $(90 - \Omega)$. Therefore, regardless of the values that R_0, Ω, and N might have, the pursuit path will always end tangent to the path of the target. If the missile is assumed to be capable of the turning that might be required to follow the path it will always catch the target by going directly in behind it. Inasmuch as countermeasures could be easily devised to destroy a missile in such a tail-chase position, this approach is considered a great disadvantage of normal pursuit navigation.

The time t corresponding to the value of R in equation (6) for a given value of ϵ is obtained from

$$t = \frac{N}{V} \int_0^{\epsilon} \frac{R}{\cos(\epsilon + \Omega)}\, d\epsilon$$

$$= \frac{N}{V(N^2 - 1)} \left\{R_0(N + \sin\Omega) - R[N + \sin(\epsilon + \Omega)]\right\} \tag{7}$$

The Cartesian coordinates (x, y) of the two-dimensional normal-pursuit path for any given values of of N, R_0, and Ω can now be obtained from equations (3), (6), and (7).

Rate of curvature.- The rate of curvature of the normal-pursuit path or the rate of turning required by a missile to follow the path is given by

$$K = \frac{d\gamma}{dt}$$

$$= \frac{d\epsilon}{dt}$$

$$= \frac{V_T}{R} \cos(\epsilon + \Omega)$$

$$= \left[\frac{V}{R_0 N \cos\Omega \left(\frac{1+\sin\Omega}{1-\sin\Omega}\right)^{N/2}}\right] \cos^2(\epsilon+\Omega) \left[\frac{1+\sin(\epsilon+\Omega)}{1-\sin(\epsilon+\Omega)}\right]^{N/2} \tag{8}$$

From equation (8), for given values of N and Ω, finite rates of curvature for particular values of ϵ are seen to be inversely proportional to R_0. Thus from the consideration of curvature a normal-pursuit path generally improves as the value of R_0 is increased. The function of N and Ω appearing in the first bracket of equation (8) can be written as

$$\frac{1}{N\cos\Omega\left(\frac{1+\sin\Omega}{1-\sin\Omega}\right)^{N/2}} = \frac{(\cos\Omega)^{N-1}}{N(1+\sin\Omega)^N} \tag{9}$$

For any particular value of $N > 1$, the value of equation (9) increases as Ω decreases from $\frac{\pi}{2}$ toward $-\frac{\pi}{2}$; therefore, better normal-pursuit paths are obtained for larger values of Ω.

The effect of N on the rate of curvature of a normal pursuit path can be seen by considering the last two factors of equation (8). These factors can be written as

$$\cos^2(\epsilon + \Omega)\left[\frac{1 + \sin(\epsilon + \Omega)}{1 - \sin(\epsilon + \Omega)}\right]^{N/2} = \frac{\left[1 + \sin(\epsilon + \Omega)\right]^{\frac{N+2}{2}}}{\left[1 - \sin(\epsilon + \Omega)\right]^{\frac{N-2}{2}}} \qquad (10)$$

If the value of N is in the range $1 < N \leqq 2$ the exponent of the denominator of equation (10) is negative or zero and the value of equation (10) is finite for all values of ϵ. For $1 < N < 2$ the curvature is zero at the end of the pursuit. If $N > 2$ the exponent of the denominator of equation (10) is positive and the curvature becomes infinite as $\epsilon \rightarrow (90 - \Omega)$. If, therefore, normal-pursuit paths that have a finite rate of curvature at all points are to be obtained, the ratio of the velocity of the missile to that of the target must not be greater than 2.

The fact that the curvature of the path becomes infinite does not necessarily exclude the higher values of N. As N increases beyond 2 the point along the path at which a given finite rate of curvature is reached moves toward the path of the target. A value of $N > 2$ can thus be determined that will cause the given rate of curvature to occur when the missile is so near the target that, for all practical purposes, it can be considered as having hit the target without having gone into a tail chase. Such values of N are comparatively large and for targets as fast as modern airplanes would require exceedingly high-speed missiles.

For example, a missile that could pull 15g normal acceleration in a steady turn would have to travel at the rate of 12,000 feet per second to come within 200 feet of a target that moved 500 feet per second along a straight course with the initial conditions $R_0 = 50{,}000$ feet, $\Omega = 0$.

It has been pointed out that for values of N in the range $1 < N < 2$ the value of the curvature is zero at the end of the pursuit. The value of ϵ for which the maximum rate of curvature occurs can be obtained by differentiating equation (8) with respect to ϵ and equating the results to zero, which gives

$$\frac{dK}{d\epsilon} = \left[\frac{V}{NR_0 \cos \Omega \left(\frac{1 + \sin \Omega}{1 - \sin \Omega}\right)^{N/2}}\right] \cos (\epsilon + \Omega) \left[\frac{1 + \sin (\epsilon + \Omega)}{1 - \sin (\epsilon + \Omega)}\right]^{N/2} [N - 2 \sin (\epsilon + \Omega)]$$

$$= 0 \tag{11}$$

or

$$\epsilon = \text{arc} \sin (N/2) - \Omega \tag{12}$$

The nondimensional distance from the target at which the maximum rate of curvature occurs can be obtained by substituting the value of ϵ from equation (12) into equation (6) and dividing the results by R_0

$$\frac{R}{R_0} = \frac{2 \cos \Omega}{\sqrt{4 - N^2}} \left(\frac{1 + \sin \Omega}{1 - \sin \Omega}\right)^{N/2} \left(\frac{2 - N}{2 + N}\right)^{N/2} \tag{13}$$

A series of curves obtained from equation (13) for values of N in the range $1 < N < 2$ and for values of Ω at intervals of 15° from -45° to 45° are shown in figure 2. By interpolation the curves can be used to estimate the distance from the target at which the maximum curvature occurs for any value of R_0 for the ranges of values covered by Ω and N.

The value of the maximum rate of curvature for $1 < N < 2$ is obtained by substituting equation (12) into equation (8)

$$K_{max} = \frac{V}{R_0 \cos \Omega \left(\frac{1 + \sin \Omega}{1 - \sin \Omega}\right)^{N/2}} \left(\frac{4 - N^2}{4N}\right)\left(\frac{2 + N}{2 - N}\right)^{N/2} \tag{14}$$

If both members of equation (14) are multiplied by R_0/V, this equation becomes of a nondimensional nature that can be evaluated and plotted for different values of Ω and N.

A plot of this kind can be used, by interpolation, to estimate the maximum rate of curvature for any values of R_0 and V for the ranges of values covered by Ω and N. In figure 3 the values obtained from equation (14) multiplied by R_0/V for values of Ω at 15° intervals from -45° to 45° and values of N in the range $1 < N \leqq 2$ have been converted to degrees and plotted against N. The dashed curve shown in figure 3 indicates the value of N that gives a minimum value of maximum rate of curvature for a given value of Ω. It is plotted from points obtained by differentiating equation (14) with respect to N, equating to zero, simplifying and solving the resulting equation for N when particular values are assigned to Ω. The derivation of the equation follows:

$$\frac{dK_{max}}{dN} = \frac{K_{max}}{2}\left\{\log\left[\left(\frac{2 + N}{2 - N}\right)\left(\frac{1 - \sin \Omega}{1 + \sin \Omega}\right)\right] - \frac{2}{N}\right\} = 0$$

or

$$\log\left[\left(\frac{2 + N}{2 - N}\right)\left(\frac{1 - \sin \Omega}{1 + \sin \Omega}\right)\right] - \frac{2}{N} = 0 \tag{15}$$

Illustrations.- In order to illustrate further the effect of the value of N and Ω on the nature of normal-pursuit paths, figure 4 is presented, which shows a number of such paths and the corresponding rates of curvature for different values of these parameters. The pursuit paths are plotted so that they can be made to apply to all values of R_0. The curves of the rate of curvature, which has been converted to degrees, require the knowledge of R_0 and V for evaluation.

Constant Navigation

Equation of path resulting from constant navigation.- When the target seeker is fixed so that the axis of its field of view makes a constant angle σ relative to the longitudinal axis of the missile the differential equations corresponding to equations (4) and (5) for normal-pursuit paths are

$$\left.\begin{aligned} \frac{d\epsilon}{dt} &= \frac{1}{R}\left[V_T \cos (\epsilon + \Omega) - V \sin \sigma\right] \\ \frac{dR}{dt} &= V_T \sin (\epsilon + \Omega) - V \cos \sigma \end{aligned}\right\} \tag{16}$$

or

$$\frac{dR}{R} = \frac{V_T \sin (\epsilon + \Omega) - V \cos \sigma}{V_T \cos (\epsilon + \Omega) - V \sin \sigma}\, d\epsilon$$

$$= \frac{\sin (\epsilon + \Omega) - N \cos \sigma}{\cos (\epsilon + \Omega) - N \sin \sigma}\, d\epsilon \tag{17}$$

If V_T is made zero, equation (17) leads to the commonly known logarithmic spiral. For other values of V_T and for positive values of σ such that $\sigma <$ arc sin 1/N, equation (17) can be integrated to give

$$R = C_1 \frac{\left[\cos\left(\epsilon + \Omega\right) - \sin E\right]^{A-1}}{\left[1 - \sin E \cos\left(\epsilon + \Omega\right) + \cos E \sin\left(\epsilon + \Omega\right)\right]^{A}} \tag{18}$$

where

$$\sin E = N \sin \sigma$$

$$\cos E = \sqrt{1 - N^2 \sin^2 \sigma}$$

$$A = \frac{N \cos}{\sqrt{1 - N^2 \sin^2 \sigma}}$$

and

$$C_1 = \frac{R_0 \left(1 - \sin E \cos \Omega + \cos E \sin \Omega\right)^{A}}{\left(\cos \Omega - \sin E\right)^{A-1}}$$

In equation (18), as the value of R ranges from R_0 to zero, the value of ϵ ranges from zero to $\left(\frac{\pi}{2} - E - \Omega\right)$. The angle at which the pursuit path and target path meet is therefore $(E - \sigma)$, which is not zero for $\sigma \neq 0$ and $N \neq 1$. Tail chases are thus avoided, but near tail chases will occur if σ is small.

The time t corresponding to R in equation (18) for a given value of ϵ is obtained from

$$t = \int_0^{\epsilon} \frac{R}{V_T \cos(\epsilon + \Omega) - V \sin \sigma} d\epsilon$$

$$= \frac{NR}{VA(A^2 - 1)\cos^3 E} \Big\{ \sin E \left[\cos(\epsilon + \Omega) - \sin E\right]$$

$$- A \cos E \left[\sin(\epsilon + \Omega) + A \cos E\right] \Big\} + C_2 \qquad (19)$$

where

$$C_2 = -\frac{NR_0}{VA(A^2 - 1)\cos^3 E} \left[\sin E (\cos \Omega - \sin E)\right.$$

$$\left. - A \cos E (\sin \Omega + A \cos E)\right]$$

Equations (18) and (19) and the substitutions given in equation (3) make it possible to compute the Cartesian coordinates of the path for given values of N, R_0, Ω, and σ.

Rate of curvature.- The rate of curvature at any point along the path is given by

$$K = \frac{d\gamma}{dt} = \frac{d\epsilon}{dt} = \frac{V}{NR} \left[\cos(\epsilon + \Omega) - \sin E\right]$$

$$= \frac{V}{NC_1} \left\{ \frac{\left[1 - \sin E \cos(\epsilon + \Omega) + \cos E \sin(\epsilon + \Omega)\right]^A}{\left[\cos(\epsilon + \Omega) - \sin E\right]^{A-2}} \right\} \qquad (21)$$

where C_1 has the same value as in equation (18).

From the expression given for C_1, finite values of the rate of curvature for particular values of ϵ appear to be inversely proportional to the initial distance R_0 for given values of N, Ω, and σ. It will be shown that two cases arise where this statement is not true.

From the exponent of the denominator of equation (21) if $1 < A < 2$, the curvature is seen to be zero when R is zero (that is, when $\cos(\epsilon + \Omega) = \sin E$).

The effect of Ω, σ, and N on the nature of pursuit paths obtained by constant navigation are so interrelated that it is difficult to isolate the effect of any one of these parameters. In view of this difficulty the important effects of certain combinations of all or part of these three parameters will be discussed.

In the factor

$$\frac{1}{C_1} = \frac{(\cos\Omega - \sin E)^{A-1}}{R_0(1 - \sin E\cos\Omega + \cos E\sin\Omega)^A} \tag{22}$$

of equation (21), if Ω, σ, and N are so related that $\cos\Omega = \sin E$, that is, if $\Omega = \pm(90 - E) \neq 0$, the value of equation (22) and hence the curvature is zero for all values of R. The pursuit path is therefore a straight line as shown in figure 1(c), cases I(a) and I(b). These two particular cases constitute the exception to the effect of R_0 mentioned previously.

Although straight line paths are ideal from the consideration of curvature, they would likely be unobtainable in actual practice by constant navigation. The value of the parameters under discussion would have to be known exactly. A small error in the evaluation of the parameters for such a path might lead to a path with very high curvature.

If the value of Ω in equation (22) is in the range $-(90 - E) < \Omega < (90 - E)$ pursuit paths of the type shown as cases II(a) and II(b) in figure 1(c) are obtained. If Ω is greater than $(90 - E)$ or less than $-(90 - E)$, pursuit paths of the type shown as cases III(a) and III(b) are obtained. The sign of the curvature in cases III(a) and III(b) is noted to be opposite to that of cases II(a) and II(b).

Combinations of the parameters Ω, σ, and N that lead to infinite rates of curvature will be shown when the expression for the maximum rate of curvature is derived.

The value of ϵ for which the maximum rate of curvature occurs is obtained by differentiating equation (21) with respect to ϵ and equating the results to zero, which gives

$$\frac{dK}{d\epsilon} = \frac{V}{NC_1}\left\{\frac{[1 - \sin E \cos (\epsilon + \Omega) + \cos E \sin (\epsilon + \Omega)]^A}{[\cos (\epsilon + \Omega) - \sin E]^{A-1}}\right\}$$

$$\times [A \cos E - 2 \sin (\epsilon + \Omega)] = 0$$

or

$$\sin (\epsilon + \Omega) = \frac{A \cos E}{2} = \frac{N \cos \sigma}{2} \tag{23}$$

for

$$0 < \frac{N \cos \sigma}{2} \leq 1$$

The value of $(\epsilon + \Omega)$ in terms of N and σ in equation (23) substituted into equation (18) gives

$$R = 2C_1 \frac{\left(\sqrt{4 - N^2\cos^2\sigma} - 2 \sin E\right)^{A-1}}{\left(2 - \sin E \sqrt{4 - N^2\cos^2\sigma} + N \cos^2\sigma\right)^A} \tag{24}$$

as the distance from the target at which the maximum rate of curvature occurs. Equation (24) divided by R_0 is shown plotted as a function of Ω for particular values of N and σ in figure 5.

The value of the maximum rate of curvature for $0 < \frac{N \cos \sigma}{2} < 1$ is obtained by substituting the value of $(\epsilon + \Omega)$ in terms of A and E found in equation (23) into equation (21), which gives

$$K_{max} = \frac{V}{4NC_1}\left[\frac{\left(2 - \sin E\sqrt{4 - A^2\cos^2 E} + A\cos^2 E\right)^A}{\left(\sqrt{4 - A\cos^2 E} - \sin E\right)^{A-2}}\right] \tag{25}$$

By use of the relations

$$\cos E = \sqrt{\frac{N^2 - 1}{A^2 - 1}}$$

and

$$\sin E = \sqrt{\frac{A^2 - N^2}{A^2 - 1}}$$

the bracketed term in equation (25) can be expressed as a function of A and N and the expression becomes

$$K_{max} = \frac{V}{4NC_1}\left(\frac{\sqrt{5A^2 - A^2N^2 - 4} + A\sqrt{A^2 - N^2}}{(2 - A)\sqrt{A^2 - 1}}\right)^A \frac{\left(\sqrt{5A^2 - A^2N^2 - 4} - 2\sqrt{A^2 - N^2}\right)^2}{(A^2 - 1)} \tag{26}$$

From the denominator of the first term in parentheses in equation (26) the maximum rate of curvature is seen to be infinite when $A = 2$. When $A = 1$ the expression becomes indeterminate but it can be shown from equation (21) that the maximum curvature is finite.

The value of N in terms of A and σ is

$$N = \frac{A}{\sqrt{1 + (A^2 - 1)\sin^2\sigma}} \tag{27}$$

From equation (27) it can be seen that for values of $\sigma \neq 0$ and $A \neq 1$ the value of N is less than A. If $A = 2$ and σ ranges from zero to $\pi/2$ the value of N ranges from 2 to 1. Thus an infinite rate of curvature can be obtained for any value of $N > 1$. Corresponding values of N and σ that give infinite rates of curvature are plotted in figure 6. The value of A as a function of σ for particular values of N is plotted in figure 7. These figures show that for a given value $\sigma \neq 0$, the upper value of N in the range of values of $N > 1$ that give finite rates of curvature (values of $A < 2$) is less than 2 and the upper value of this range decreases as σ increases.

For values of $A > 2$ the rate of curvature is infinite at the end of the pursuit as can be seen by considering the exponents in the following limit:

Limit

$$\epsilon \rightarrow \left[\frac{\pi}{2} - (\epsilon + \Omega)\right]$$

$$\frac{V}{NC_1} \frac{[1 - \sin E \cos (\epsilon + \Omega) + \cos E \sin (\epsilon + \Omega)]^A}{[\cos (\epsilon + \Omega) - \sin E]^{A-2}} \tag{28}$$

In the range of values of $A > 2$ the point along the pursuit path at which a given rate of curvature occurs moves toward the path of the target as A increases; therefore, A can be made large enough to bring the missile so near the target before any appreciable turning is required that it can be considered to have hit the target.

The value in degrees per unit of time of the maximum rate of curvature multiplied by R_0 and divided by V for particular values of N and σ is plotted in figure 8 as a function of Ω for values in the range $-(90 - E) < \Omega < (90 - E)$. In figure 8 for values of N near 1.2 small maximum rates of curvature corresponding to a wide range of values of Ω and σ occur; as N increases, the range of values of Ω and σ that give finite rates of curvature decrease. For values of Ω outside of the range $-(90 - E) < \Omega < (90 - E)$, the rate of curvature always increases as R decreases.

Illustrations.- Figures 9 and 10 show some illustrative pursuit paths and the corresponding rates of curvature obtained from constant navigation for various values of N, Ω, and σ. The curves are plotted in the same nondimensional form as those of normal pursuit.

Proportional Navigation

Unfortunately, the differential equations of the paths resulting from proportional navigation can be integrated in closed form for only special cases. The general treatment given the equations for normal pursuit and constant navigation cannot be given. The case for a 2:1 rate of navigation can be integrated to yield R, x, and K but not t and y. This case is therefore discussed and conclusions for low rates of navigation can be drawn from the results. Solutions for other rates of navigation obtained by the differential analyzer will be presented and discussed.

Although it has been assumed for this discussion that target-seeking aircraft are not rendered dynamically unstable by the target seeker, this assumption may be unjustifiable, especially for high rates of proportional navigation.

<u>Equation of path resulting from 2:1 proportional navigation</u>.- The differential equations of the path resulting from an $n:1$ rate of proportional navigation are

$$\left.\begin{aligned} \frac{d\epsilon}{dt} &= \frac{1}{R}\left[V_T \cos\left(\epsilon + \Omega\right) - V \sin\left(n - 1\right)\epsilon\right] \\ \frac{dR}{dt} &= \left[V_T \sin\left(\epsilon + \Omega\right) - V \cos\left(n - 1\right)\epsilon\right] \end{aligned}\right\} \qquad (29)$$

or

$$\frac{dR}{R} = \frac{\sin\left(\epsilon + \Omega\right) - N \cos\left(n - 1\right)\epsilon}{\cos\left(\epsilon + \Omega\right) - N \sin\left(n - 1\right)\epsilon}\, d\epsilon \qquad (30)$$

Equation (30) can be easily handled by numerical methods for any value of n. The range of ϵ is from zero to the solution of

$$\cos\left(\epsilon + \Omega\right) - N \sin\left(n - 1\right)\epsilon = 0 \qquad (31)$$

The angle at which the pursuit path and target path meet is

$$\frac{\pi}{2} - (n\epsilon + \Omega) \tag{32}$$

Tail chases are therefore not encountered.

When $n = 2$ equation (30) becomes

$$\frac{dR}{R} = \frac{\sin (\epsilon + \Omega) - N \cos \epsilon}{\cos (\epsilon + \Omega) - N \sin \epsilon} d\epsilon \tag{33}$$

which can be integrated to give

$$R = R_0 \left\{ \left[\frac{\cos (\epsilon + \Omega) - N \sin \epsilon}{\cos \Omega} \right]^{N^2-1} e^{-2N\epsilon\cos\Omega} \right\}^{\frac{1}{N^2+1+2N\sin\Omega}} \tag{34}$$

Equation (31) now becomes

$$\cos (\epsilon + \Omega) - N \sin\epsilon = 0 \tag{35}$$

for which the solution is

$$\epsilon = \text{arc tan} \left(\frac{\cos \Omega}{N + \sin \Omega} \right) \tag{36}$$

The angle at which the pursuit path and target path meet is, from equation (32) and (36),

$$\frac{\pi}{2} - \text{arc tan} \left[\frac{2N + (N^2 + 1) \sin \Omega}{(N^2 - 1) \cos \Omega} \right] \tag{37}$$

The value of t corresponding to R is given by

$$t = \frac{R_0 N}{V} \int_0^{\epsilon} \left\{ \frac{e^{-2N\epsilon\cos\Omega}}{(\cos\Omega)^{N^2-1}\left[\cos(\epsilon+\Omega) - N\sin\epsilon\right]^{2(1+N\sin\Omega)}} \right\}^{\frac{1}{N^2+1+2N\sin\Omega}} d\epsilon \tag{38}$$

which cannot be integrated by simple methods.

Rate of curvature resulting from 2:1 proportional navigation.- The rate of curvature at any point along the path is given by

$$K = \frac{d\gamma}{dt} = n\,\frac{d\epsilon}{dt} = \frac{2V}{NR}\left[\cos(\epsilon+\Omega) - N\sin\epsilon\right]$$

$$= \frac{2V}{NR_0}\left\{(\cos\Omega)^{N^2-1}\left[\cos(\epsilon+\Omega) - N\sin\epsilon\right]^{2(1+N\sin\Omega)} e^{2N\epsilon\cos\Omega}\right\}^{\frac{1}{1+N^2+2N\sin\Omega}} \tag{39}$$

As in the case of normal pursuit, for given values of all other parameters, the rate of curvature is inversely proportional to the initial distance R_0 between the missile and target.

Since the value of R in equation (34) is zero when equation (35) is satisfied, the corresponding value of K given by equation (39) is also zero, provided that $(1 + N \sin \Omega) > 0$. Differentiating equation (39) with respect to ϵ and simplifying gives

$$\frac{dK}{d\epsilon} = -\frac{4V}{NR} \sin (\epsilon + \Omega) \tag{40}$$

from which, if $\Omega \geqq 0$, the slope of the curve representing the rate of curvature is seen to be negative for all values of ϵ in the range

$$0 \leqq \epsilon \leqq \text{arc tan} \left(\frac{\cos \Omega}{N + \sin \Omega}\right)$$

The maximum rate of curvature is therefore at the initial point of the pursuit path and continuously decreases to zero as $R \rightarrow 0$. The maximum value can be reduced by launching the missile so that it leads the target.

If $N \sin \Omega + 1 = 0$ or $\sin \Omega = -\frac{1}{N}$ (Ω in fourth quadrant) equation (39) reduces to

$$K = \frac{2V \sqrt{N^2 - 1}}{N^2 R_0} e^{2\epsilon/\sqrt{N^2-1}} \tag{41}$$

which is always finite for values of $N > 1$. In this case the greatest rate of curvature occurs at the end of the pursuit path but as N increases, the terms $\sqrt{N^2 - 1}/N^2$ and $2/\sqrt{N^2 - 1}$, the final value of ϵ and, consequently, the rate of curvature at any point along the path all decrease.

If $(N \sin \Omega + 1) < 0$, the rate of curvature is infinite at the end of the path but, as in the case of normal pursuit and constant navigation, the point at which a given rate of curvature occurs moves toward the path line of the target as N increases. If N is sufficiently large the missile will be near enough to the target before a specified rate of turning is required to be considered to have hit the target; thus, for any fixed values of R_0 and Ω, the path obtained from 2:1 proportional navigation is seen to improve as N increases.

Differential-analyzer solutions.- Some of the results of differential-analyzer solutions of the differential equations for porportional navigation, plotted in the same nondimensional form as those for normal pursuit and constant navigation, are presented in figures 11 to 13. Some of the curves in these figures, especially those for which the rate of change in curvature becomes great, do not extend to the path line of the target because, during the solution of the equations, one part of the operation of the differential analyzer machine was controlled manually. When the rate of change in curvature became high, operation of this control became difficult and in extreme cases, impossible. The portion of the curves shown should be correct to within ±5 percent.

Comparisons of flight paths and corresponding rates of curvature for $\Omega = 0^{\circ}$ and for particular values of N and n are shown in figure 11(a). Figure 12 shows corresponding curves for $\Omega = -45^{\circ}$ and figure 13 shows a comparison of flight paths and the corresponding rates of curvature for $\Omega = 0^{\circ}$, $N = 2$ and different values of the proportionality factor n. These figures show clearly the favorable effect on the curvature of increasing the value of N.

In figure 13 the greatest initial rate of curvature corresponds to the greatest rate of navigation but the greater the rate of navigation, the faster the rate of curvature beyond the initial point approaches zero. If the rate of navigation were made infinite, the rate of curvature would be infinite for the first instant of time and would be zero thereafter.

Theoretically, the curvature corresponding to any rate of proportional navigation can be made zero at all points along the path if the missile is made to lead the target by an angle

$$\text{arc sin}\left(\frac{\cos\,\Omega}{N}\right)$$

In actual practice this angle might be approximated and the curvature at all points along the path made very small, particularly for small rates of navigation.

In view of the small range of values of N that were found to yield finite rates of curvature for normal pursuit and constant navigation it appears that proportional navigation is by far the best of the methods considered for installing a target seeker in a missile.

Line-of-Sight Navigation

Equation of line-of-sight path.- Although line-of-sight navigation applies to cases where the missile is kept either between a moving control station and the target or a stationary control station and the target, the later case is considered in this discussion.

Referring to figure 1(e) the initial position of the missile and the control station are assumed to be at the origin of the Cartesian coordinate system and the target is assumed to travel in a positive direction along the line $x = a$. If equal expressions of time are equated the differential equation

$$N\,\frac{dy_T}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{42}$$

is obtained and, from the similarity of triangles determined by the points $(0,0)$, $(a,0)$, (a,y_T) and $(0,0)$, $(x,0)$, (x,y), the relation

$$y_T = \frac{ay}{x} = (R_0\,\cos\,\Omega)\frac{y}{x} \tag{43}$$

is obtained. Eliminating y_T from equations (42) and (43) gives the differential equation of the path in Cartesian coordinates,

$$NR_0 \cos \Omega \, (Px - y) = x^2\sqrt{1 + P^2} \tag{44}$$

where

$$P = \frac{dy}{dx}$$

Equation (44) cannot be integrated by simple methods but can be reduced to a function of elliptic integrals, so that it can be evaluated with the aid of tables or can be reduced to a simple parametric form for numerical evaluation. Both reductions follow.

After differentiating equation (44) with respect to x and simplifying, the results can be written as

$$\frac{dx}{dP} + \frac{Px}{2(1 + P^2)} = \frac{NR_0 \cos \Omega}{2\sqrt{1 + P^2}} \tag{45}$$

which can be reduced to the quadrature

$$x \sqrt[4]{1 + P^2} = \frac{NR_0 \cos \Omega}{2} \int_{P_0}^{P} \frac{dP}{\sqrt[4]{1 + P^2}} \tag{46}$$

Substituting

$$P = \frac{\sin \theta \sqrt{1 + \cos^2\theta}}{\cos^2 \theta} \tag{47}$$

into equation (46) and simplifying gives

$$x = NR_0 \cos \Omega \left(\sin \theta \sqrt{1 + \cos^2\theta} - \sqrt{2} \cos \theta \int_{\theta_0}^{\theta} \sqrt{1 - \frac{1}{2} \sin^2\theta} \, d\theta + \frac{\cos \theta}{\sqrt{2}} \int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{1 - \frac{1}{2} \sin^2\theta}} \right) \tag{48}$$

Equation (48) expresses the value of x as a function of elliptic integrals of the first and second kinds. The corresponding value of y can be obtained from equations (44), (47), and (48).

Solving equations (44) and (46) for x and y in terms of integral functions of P gives

$$\left.\begin{aligned} x &= \frac{NR_0 \cos \Omega}{2\sqrt[4]{1 + P^2}} \int_{P_0}^{P} \frac{dP}{\sqrt[4]{1 + P^2}} \\ y &= Px - \frac{NR_0 \cos \Omega}{4} \left(\int_{P_0}^{P} \frac{dP}{\sqrt[4]{1 + P^2}} \right)^2 \end{aligned}\right\} \tag{49}$$

These expressions can be evaluated by numerical methods to yield the coordinates of the path.

Rate of curvature.- The rate of curvature as a function of P is

$$K = \frac{1}{NR_0 \cos \Omega} \frac{2V}{\sqrt{1 + P^2}\left(\sqrt{1 + P^2} - \frac{Px}{NR_0 \cos \Omega}\right)} \tag{50}$$

where P_0 has the same sign as Ω. The rate of curvature is dependent upon the inverse value of N and the inverse value of $R_0 \cos \Omega = a$. More favorable paths would therefore be obtained for larger values of $\Omega > 0$ and for larger values of N. An infinite rate of curvature requires that

$$\frac{Px}{NR_0 \cos \Omega} = \sqrt{1 + P^2}$$

or

$$\frac{x}{Na} = \sqrt{1 + \frac{1}{P^2}} \tag{51}$$

The value of the right-hand side of equation (51) is always greater than unity and in the left-hand side, as x ranges from 0 to a, the value of x/Na ranges from 0 to $1/N$. Hence, for values of $N > 1$ the value of x/Na is always less than unity and equation (51) is not valid. The curvature is therefore finite if $N > 1$ and $R_0 \neq 0$.

Illustrative examples.- Figure 14 presents line-of-sight paths and the corresponding rates of curvature for $\Omega = -45^\circ$, 0°, and 45°. The coordinates of the pursuit paths have been divided by Na and rates of curvature have been converted to degrees and multiplied by Na so that they are of a nondimensional nature. They can be used to derive paths corresponding to the values of $\Omega = 0^\circ$ and $\pm 45^\circ$ for any values of N and a.

EFFECT OF ANGLE OF ATTACK AND SIDESLIP

It will be recalled that in all of the pursuit paths that have been discussed no allowance has been made for aerodynamic forces that would act on a missile if it attempted to fly any of the derived paths. In the case of an aircraft that makes turns by banking, the principal factor arising from aerodynamic forces is an increase in angle of attack required to compensate for the loss in lift due to banking. With a target-seeking missile, as the angle of bank and corresponding angle of attack are increased, the axis of the field of view of the target seeker will point above and ahead of the target. The target seeker will therefore give signals for nosing down and turning back toward the target. Consequently, the missile will never get into the exact attitude for making the required turn but will continue, for the greater part of its course, along a path of less curvature than the theoretical path.

While the missile is in a banked attitude and at an insufficient angle of attack to give the required lift, a slight amount of inward sideslip will result. This factor, however, may generally be considered negligible compared with the outward deviation from the theoretical path.

If the missile is designed to make horizontal turns without banking, the important factor arising from dynamic forces is that of positive sideslip or skidding which again tends to decrease the curvature of the path.

When either the effect of an increase in angle of attack due to turning or the effect of skidding due to turning is properly taken into account, the analytical expressions for the flight path generally become unwieldy and step-by-step methods must be employed to obtain the coordinates of the path. Because of this difficulty the normal-pursuit and constant-navigation paths that have finite rates of curvature can, by proper choice, be used to best advantage by considering them as boundaries of actual two-dimensional paths that might be obtained by constant navigation. For example, suppose that, for a given missile with the target seeker fixed for normal pursuit, it is known that $1 < N < 2$ and, at the point of maximum curvature of the path, the angle between the theoretical and actual direction of the axis of the target seeker is β_{max}. If the target seeker is then rearranged for constant navigation with $\sigma = \beta_{max}$, a reasonable assumption is that the actual path will be between the normal pursuit path and the path obtained by constant navigation with $\sigma = \beta_{max}$. In fact, the actual path would be expected to be near the constant-navigation path at the beginning of the course, to approach the normal pursuit path at the point of maximum curvature, and then to approach the constant navigation path toward the end of the pursuit. Thus, if tail chases are to be avoided, the point of maximum curvature should be near the beginning of the course so that the missile would have time to recover from maximum deviation from the theoretical path or the value of σ should be made so large that the path never approaches a normal pursuit path.

In the case of proportional navigation for $n \geqq 2$ and for values of Ω in the range that cause the maximum curvature to occur at the initial point of the pursuit path, the effect of increased angle of attack or of sideslip can be made insignificant by launching the missile so that it leads the target by the proper amount.

In the case of normal pursuit for $N > 2$, constant navigation for $A > 2$ and proportional navigation for $n \geqq 2$ and values of Ω that give infinite curvature at the end of the pursuit path, if N is made large enough to require little turning before the missile is very near the target, effects of angle of attack and sideslip are negligible.

With line-of-sight navigation, the target seeker is not mounted in the missile. The orientation of the missile is therefore not necessarily a factor in determining the navigation. In other words the path of the missile is unaffected by angle of attack or sideslip for this case.

CONCLUSIONS

A discussion was given of four possible methods of using a target seeker to make a missile pursue a moving target, namely, normal pursuit, constant navigation, proportional navigation, and line-of-sight navigation. The following conclusions are drawn from the results:

1. Normal pursuit.- Normal pursuit paths with finite curvatures are obtained only when the ratio N of the velocity of the missile to that of the target is in the range $1 < N \leqq 2$. For given values of N and the initial angular position Ω of the target, the curvature at any point of the path is inversely proportional to the initial distance R_0 between the missile and the target. For given values of R_0 and N the path improves as Ω increases from the smallest value it can have to the largest value it can have $\left(-\frac{\pi}{2} < \Omega < \frac{\pi}{2}\right)$. Regardless of the values of N, R_0, and Ω the path ends up tangent to the path of the target.

For values of N in the range $1 < N < 2$ the value of the curvature is zero when $R = 0$ and has a maximum value corresponding to some value of $R \neq R_0$. If the maximum curvature occurs near the end of the pursuit the effect of increased angle of attack or of sideslip might prevent the missile from making the prescribed turn and thereby cause it to miss the target. If the missile does make the turn it will end its pursuit in a tail chase where it is most vulnerable to destruction by crewmen of the target.

For values of $N > 2$ the curvature is infinite when $R = 0$. If N is made large enough, pursuit paths are obtained that have very little curvature until the missile is so near the target that it can be considered to have hit the target. With such a path tail chases are avoided and effects of increased angle of attack or of sideslip are insignificant.

2. Constant navigation.- Pursuit paths obtained from constant navigation have finite curvature only if the value of $A = \dfrac{N \cos \sigma}{\sqrt{1 - N^2 \sin^2 \sigma}}$, where σ is the constant angle of lead, is in the range $1 < A < 2$ and $N < 2$. For given values of N, Ω, and σ the value of the curvature at any point along the path is generally inversely proportional to R_0. If the relation between N, Ω, and σ is such that $\Omega = \pm(90 - E)$, straight-line paths are obtained.

For values of A in the range $1 < A < 2$ the value of the curvature is zero when $R = 0$ and has a maximum value corresponding to some value of $R \neq R_O$. The effect of increased angle of attack or of sideslip is the same as for normal pursuit. Tail chases might occur if σ is small.

For values of $A > 2$ the curvature is infinite when $R = 0$ but, if A is made large enough, pursuit paths are obtained that have very little curvature before the missile is near enough the target that it can be considered to have hit the target. Tail chases are avoided by such a path and the effect of increased angle of attack or of sideslip are insignificant.

3. Proportional navigation.- Proportional navigation appears to be by far the best of the methods considered for using a target seeker installed in a missile. With a 2:1 rate of navigation pursuit paths with finite curvature are obtained where $(N \sin \Omega + 1) \geq 0$. For given values of N and Ω the curvature is inversely proportional to R_O. For given values of R_O and Ω the paths improve as N increases.

For rates of navigation higher than 2:1 the paths improve as N increases. The maximum curvature generally occurs at the initial point of the pursuit path and can be reduced by launching the missile so that it slightly leads the target. Higher rates of navigation may have adverse effects on the stability of the missile, otherwise they give better paths.

Tail chases are not encountered for any value of $N > 1$ and the effect of increased angle of attack or of sideslip is not significant so long as the curvature is reasonably small.

4. Line-of-sight navigation.- The curvature of line-of-sight paths is generally finite. It is dependent upon N, R_O, and Ω and is more favorable for larger values of $\Omega > 0$, and for larger values of N. Angle of attack and sideslip have no effect on the path because the target seeker is not mounted in the missile.

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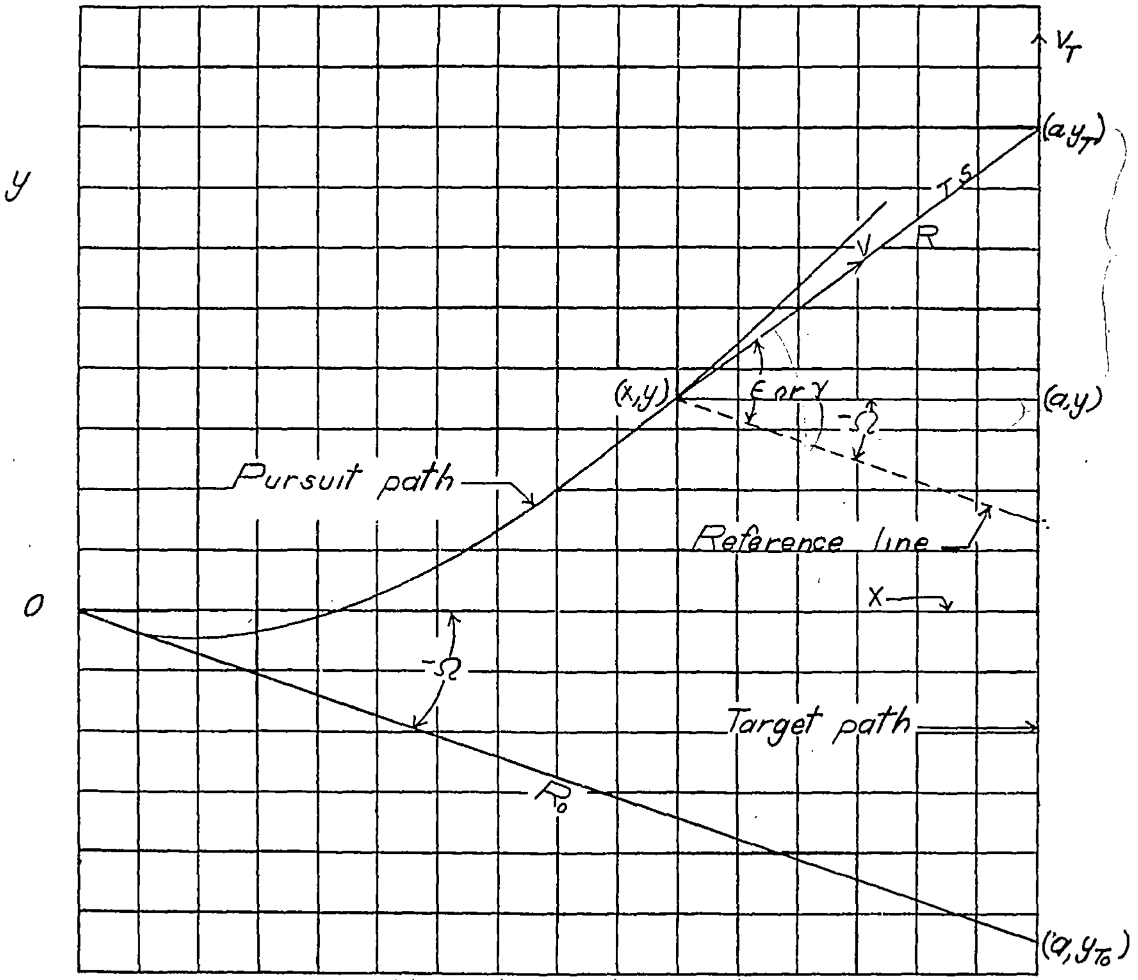

(a) Normal-pursuit navigation.

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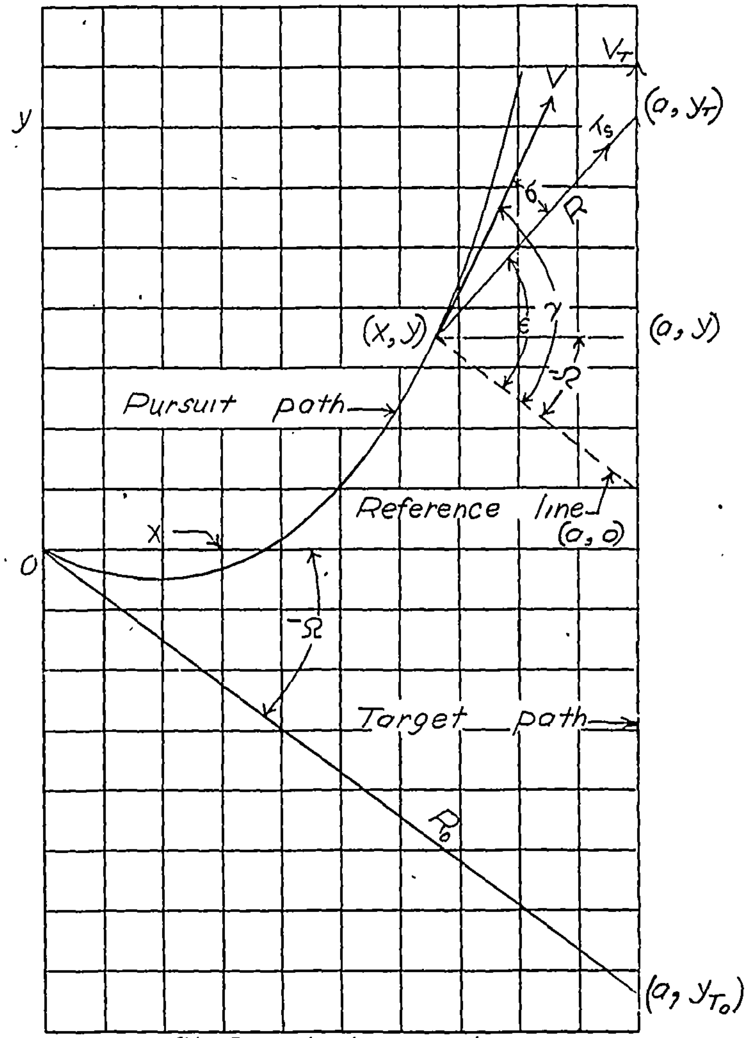
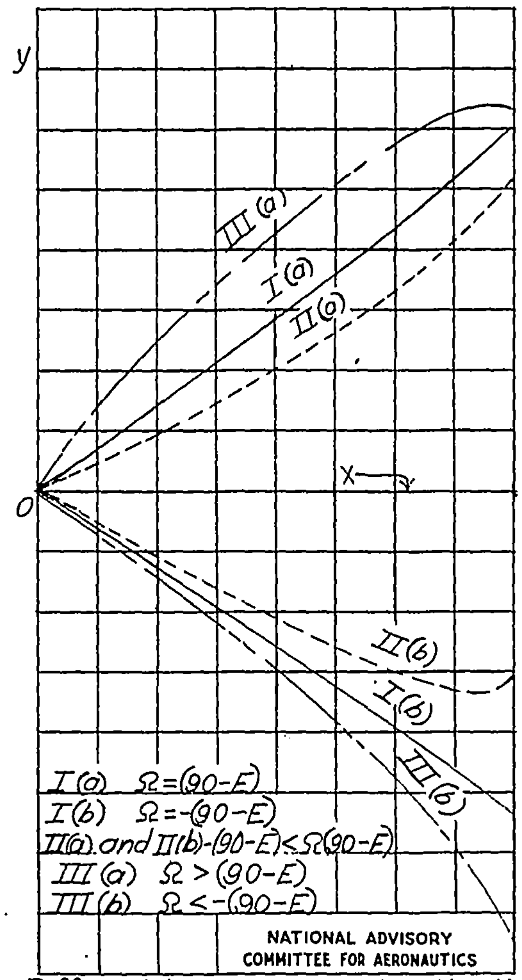
Figure 1.- Geometrical setup for paths of target-seeking missiles.

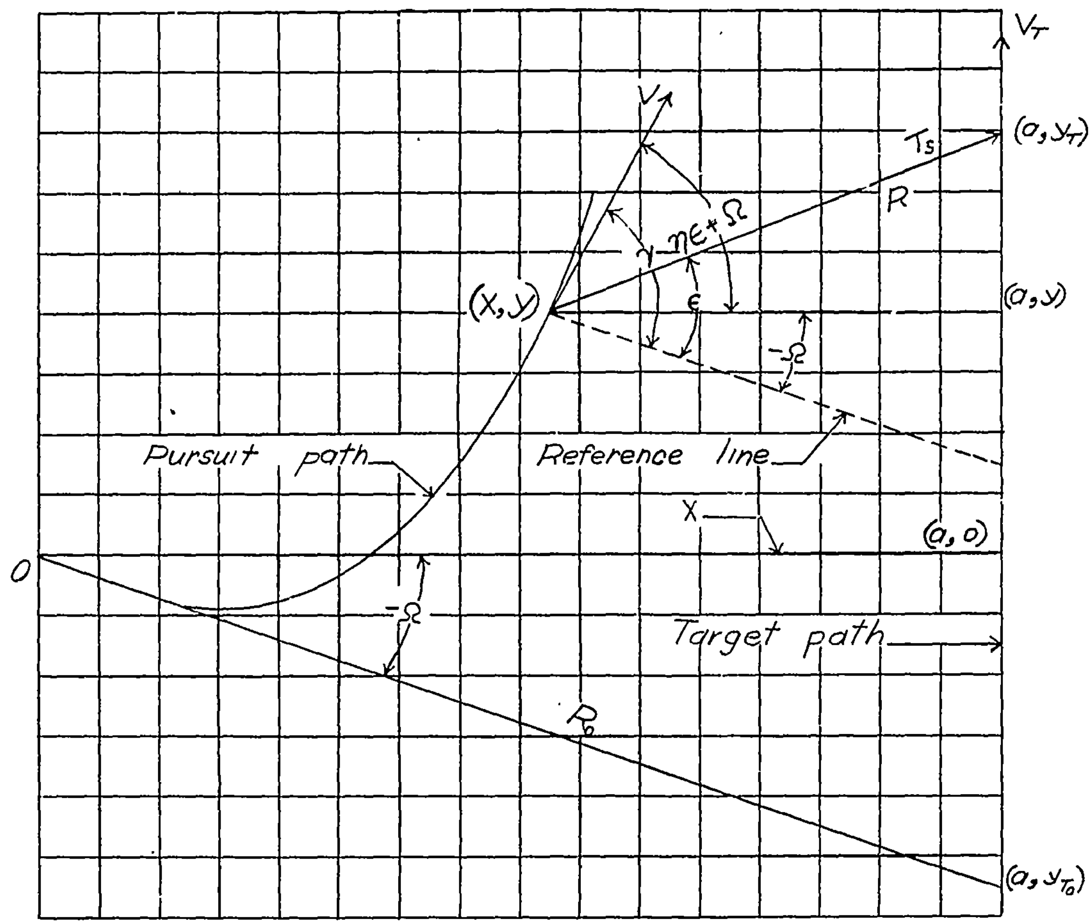


(b) Constant navigation.



(c) Different types of pursuit paths that can be obtained with constant navigation for different relations between Ω and E.

Figure 1.- Continued.



(d) Proportional navigation.

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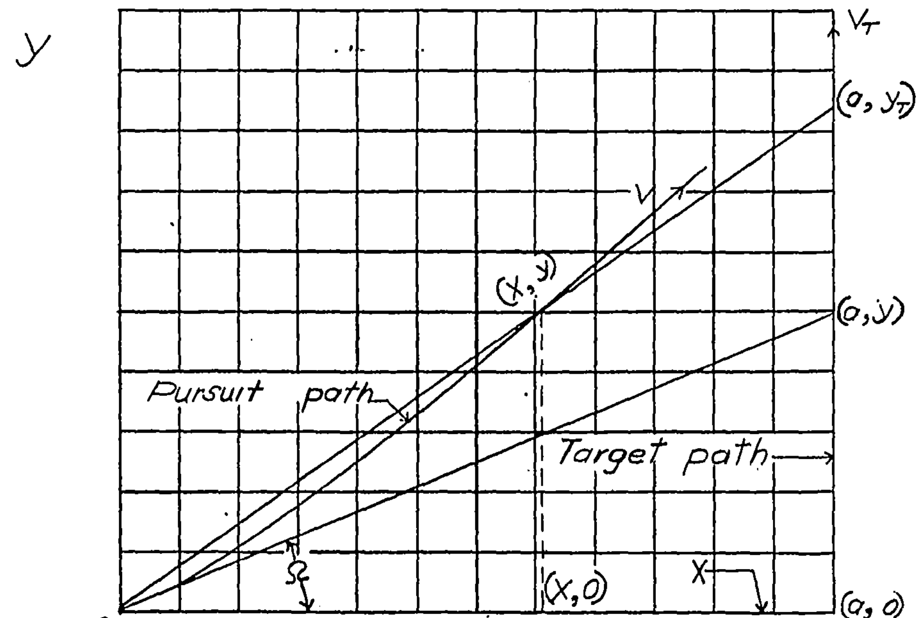
Figure 1.- Continued.



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(e) Line-of-sight navigation.

Figure 1.- Concluded.

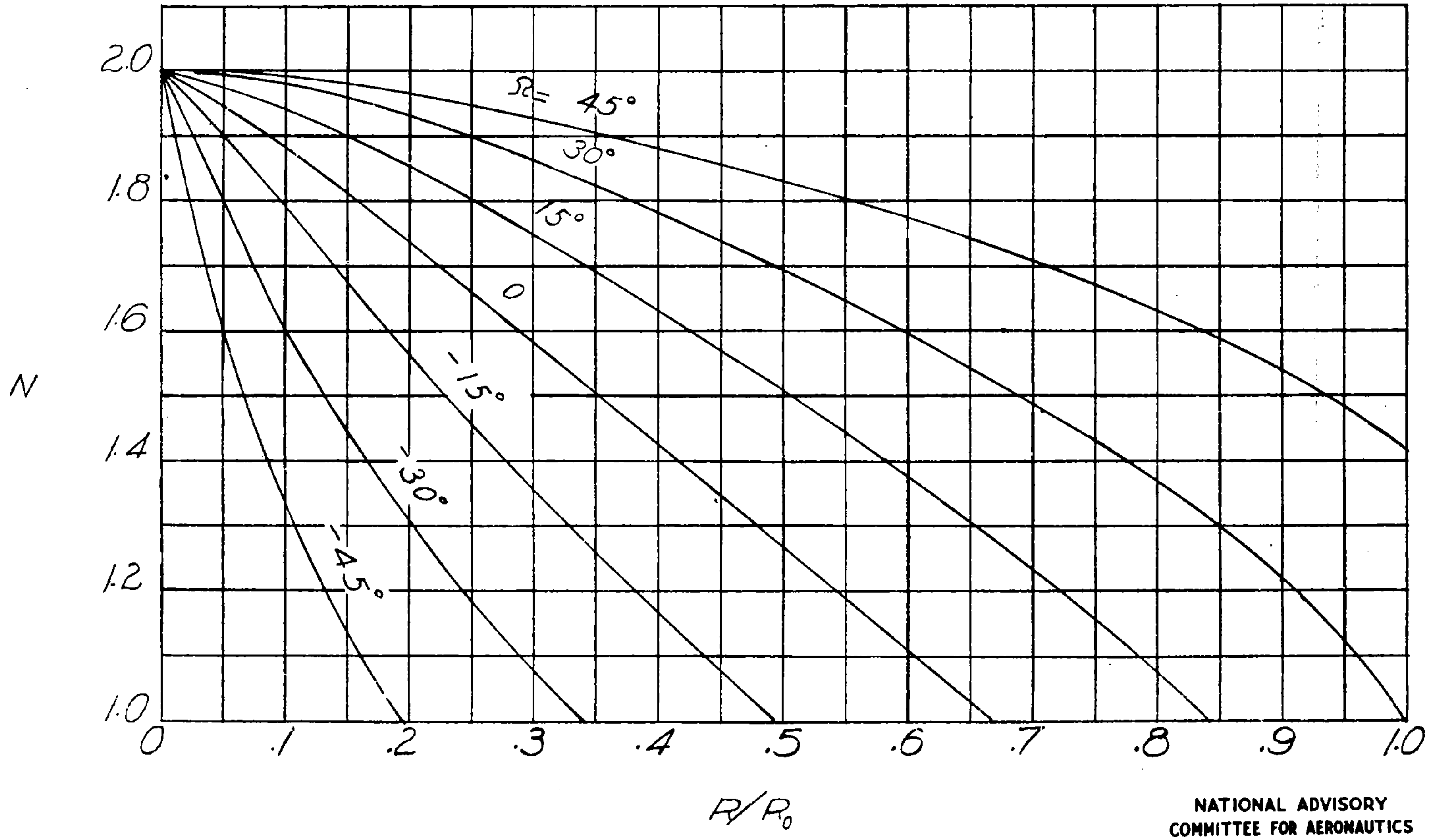


Figure 2.– Variation of R/R_0 with N for which maximum curvature is reached for some particular values of Ω. Normal-pursuit navigation.

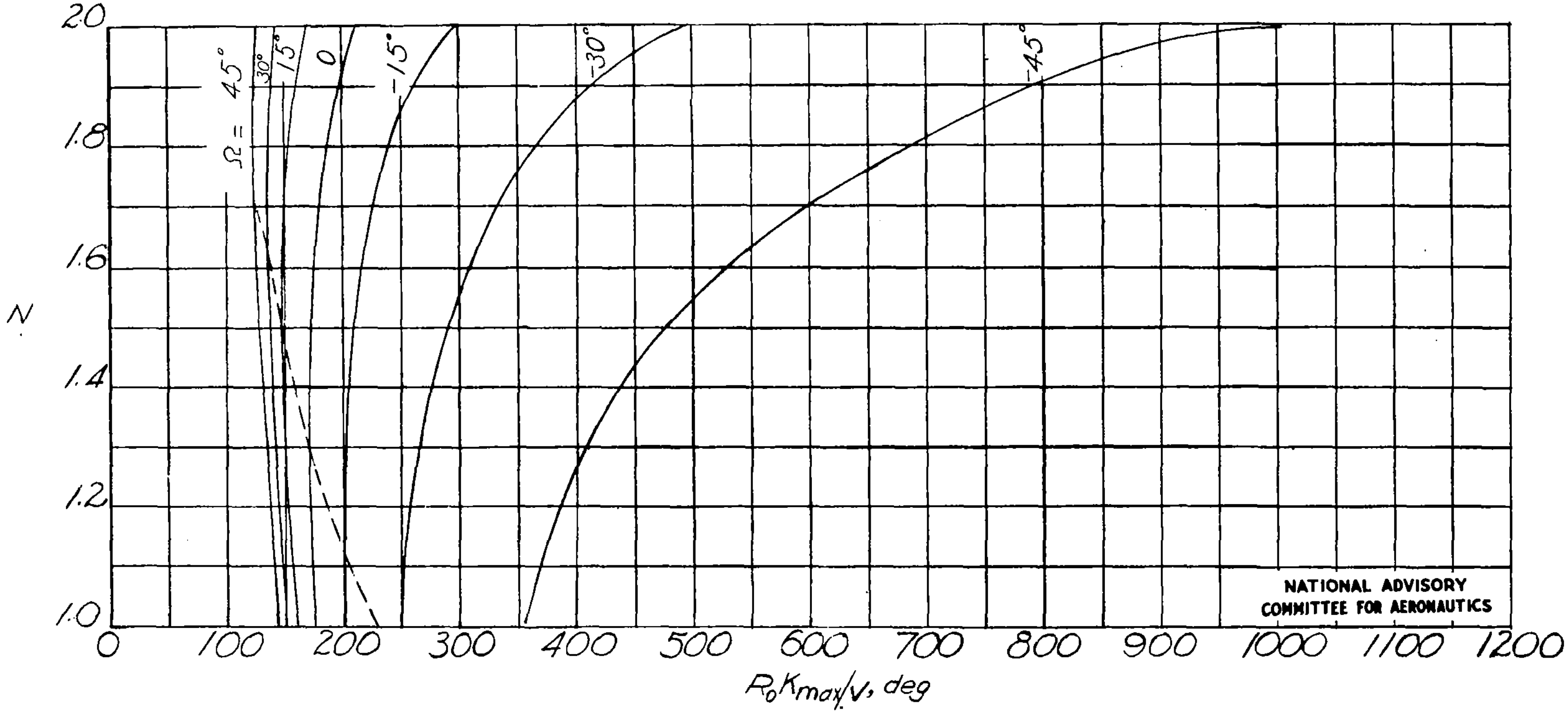
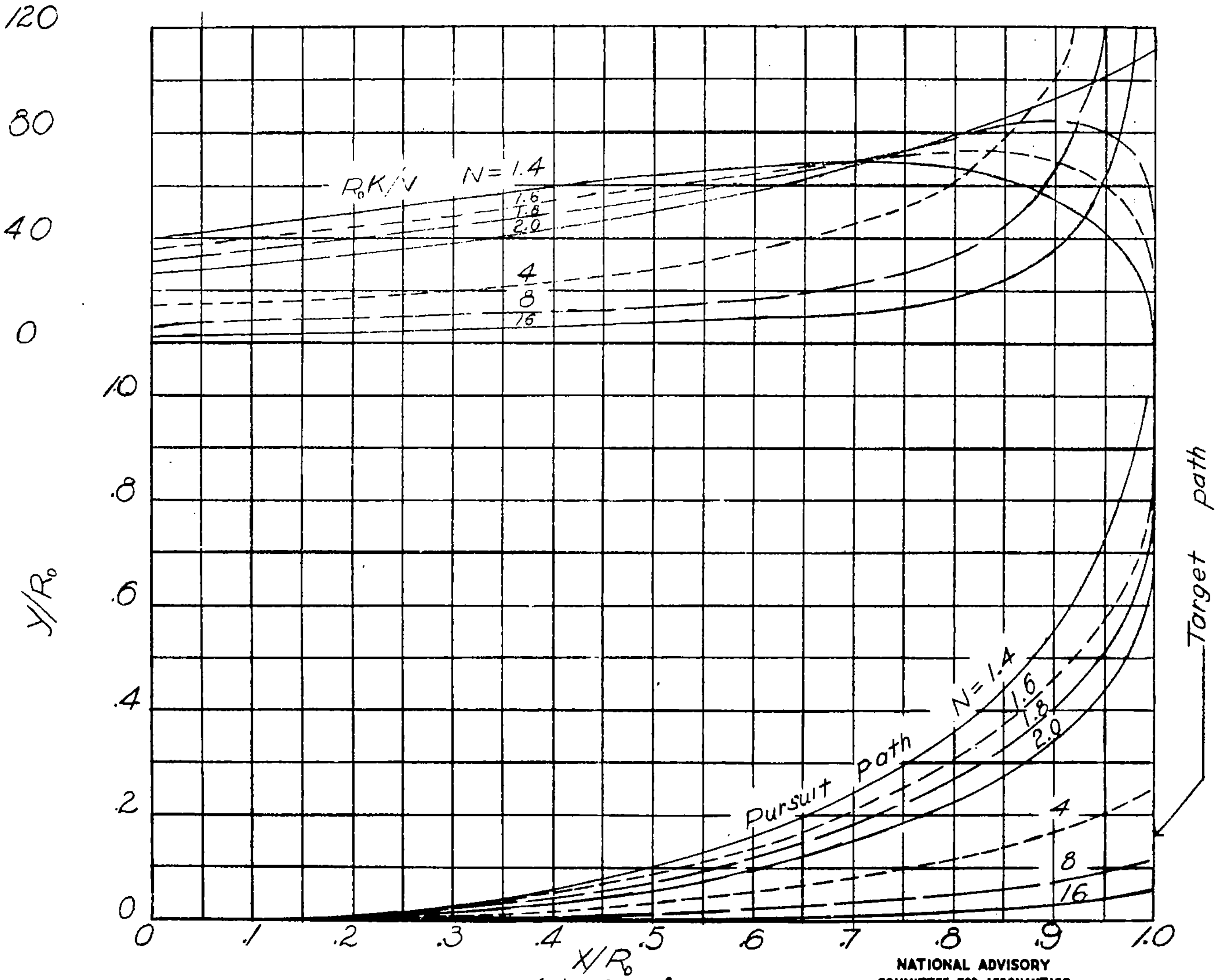


Figure 3.– Variation of R_0K_{max}/V with N for particular values of Ω, normal-pursuit navigation. (Dashed line indicates corresponding values of N and Ω for which maximum curvature is a minimum.)



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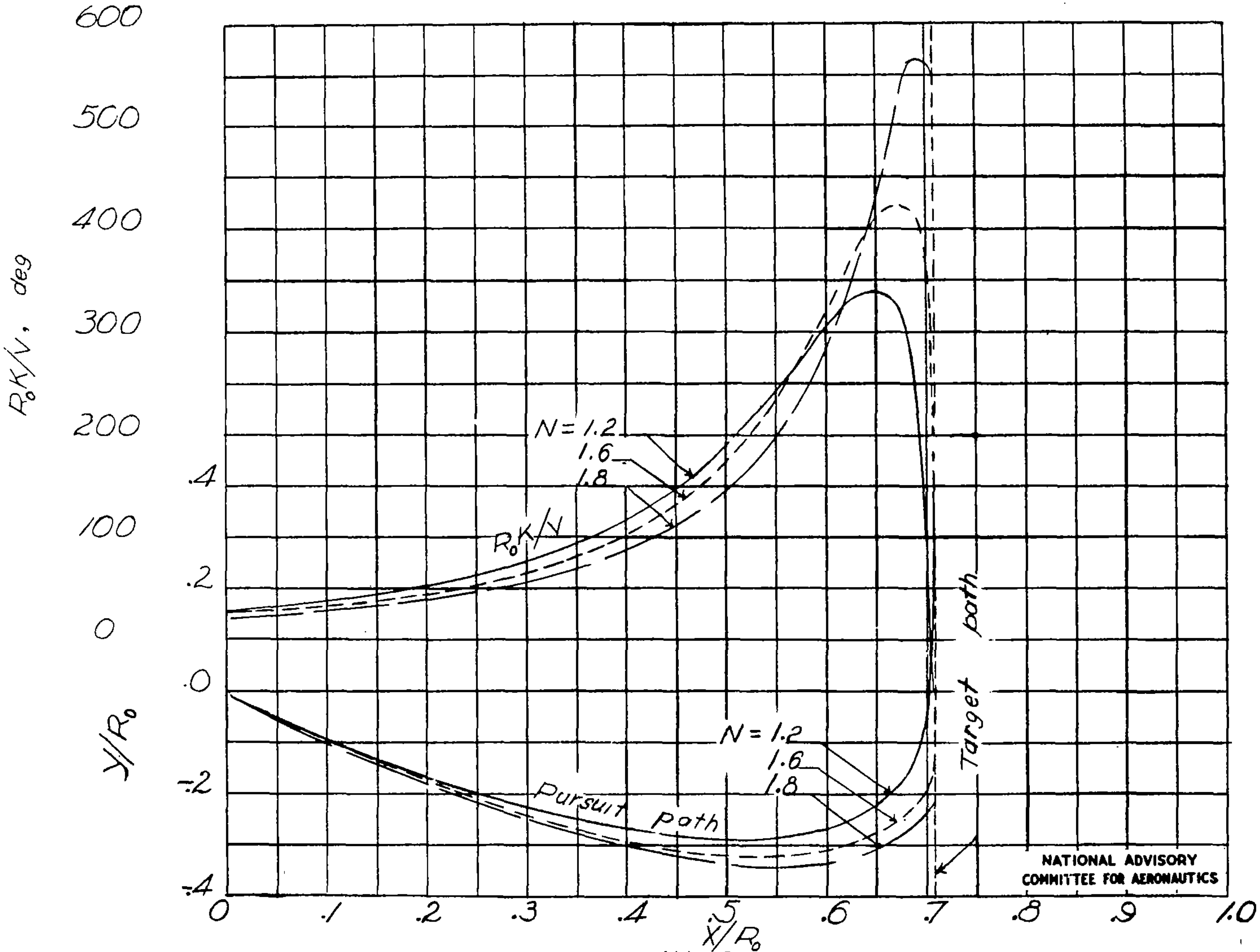
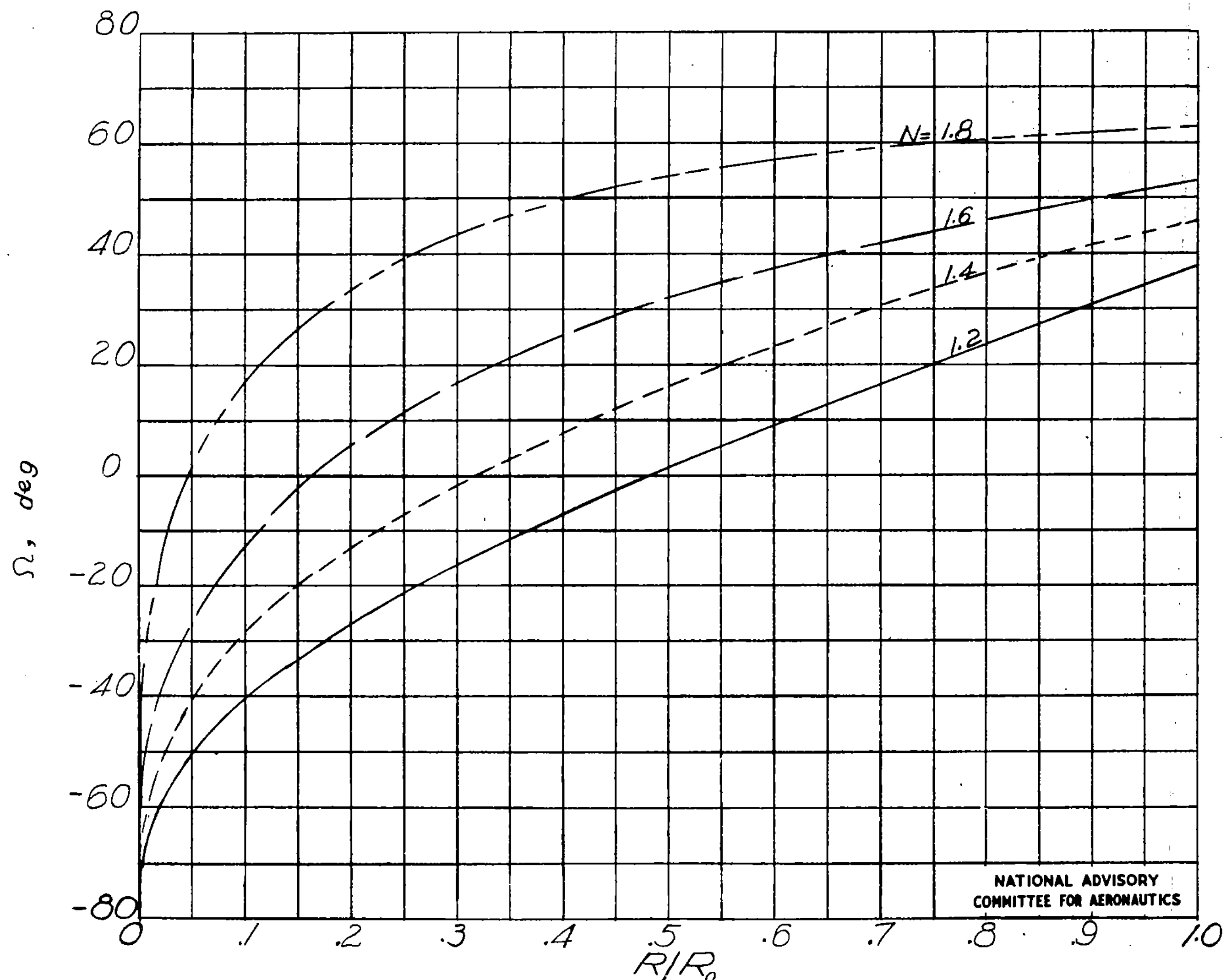
(a) $\Omega = 0°$.

Figure 4.- Comparison of pursuit paths and corresponding rates of curvature obtained by normal pursuit navigation for different values of N and Ω.



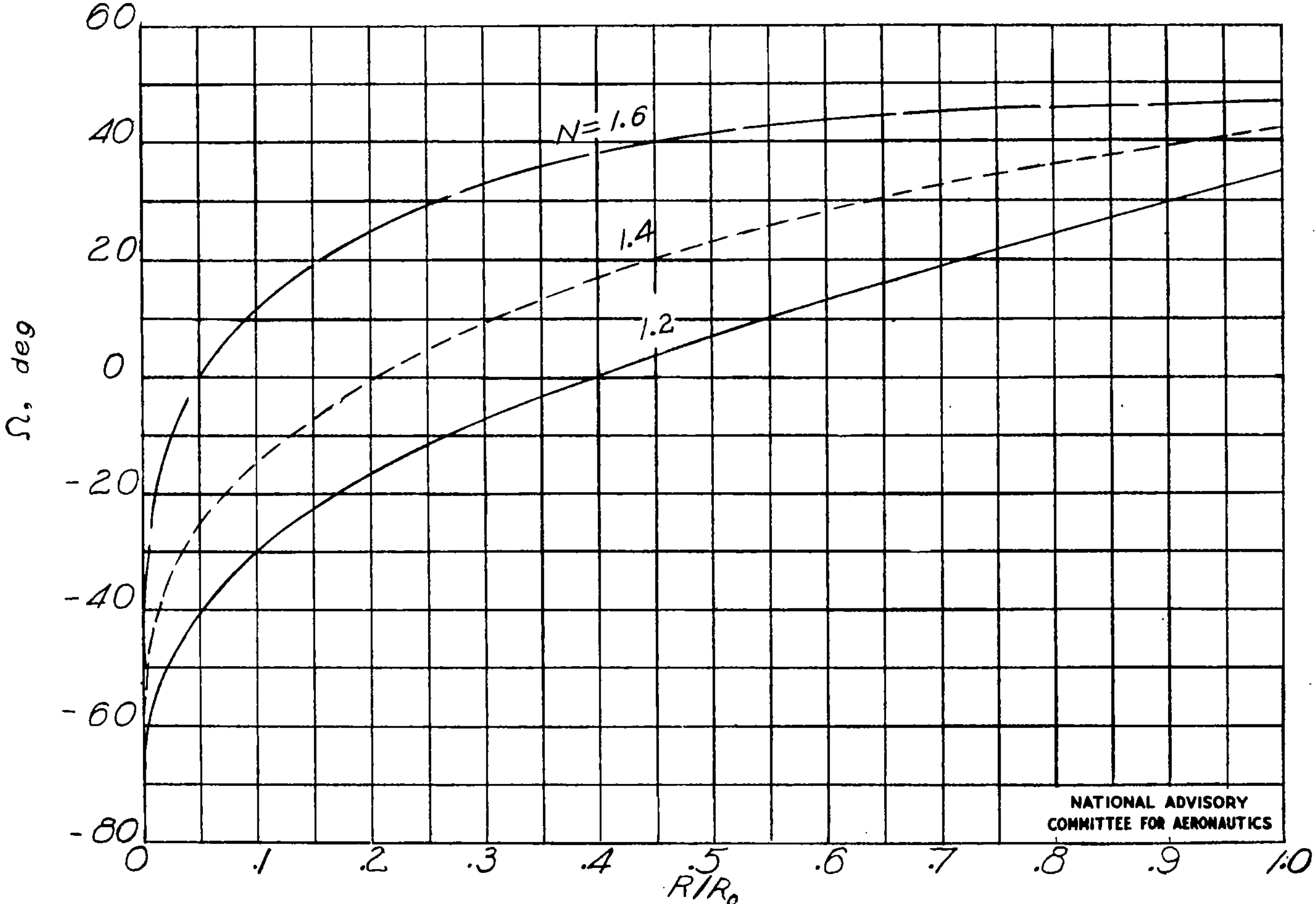
(b) $\Omega = -45°$

Figure 4-Concluded.



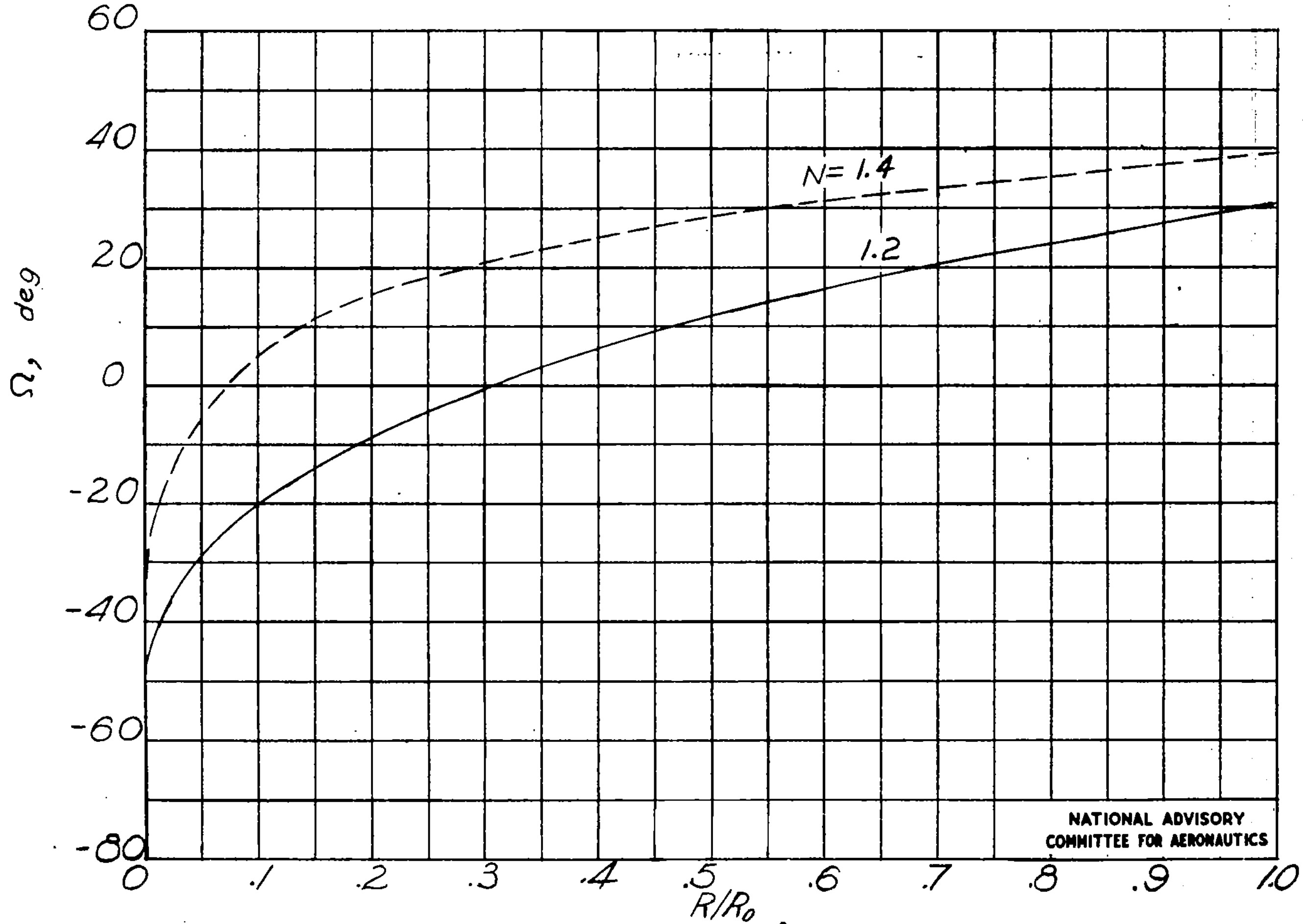
(a) $\sigma = 10°$

Figure 5.— Variation of R/R_0 with Ω for which maximum curvature is reached for some particular values of N and σ. Constant navigation.



(b) $\sigma = 20°$

Figure 5.- Continued.



(c) $\sigma = 30°$.

Figure 5.- Concluded.

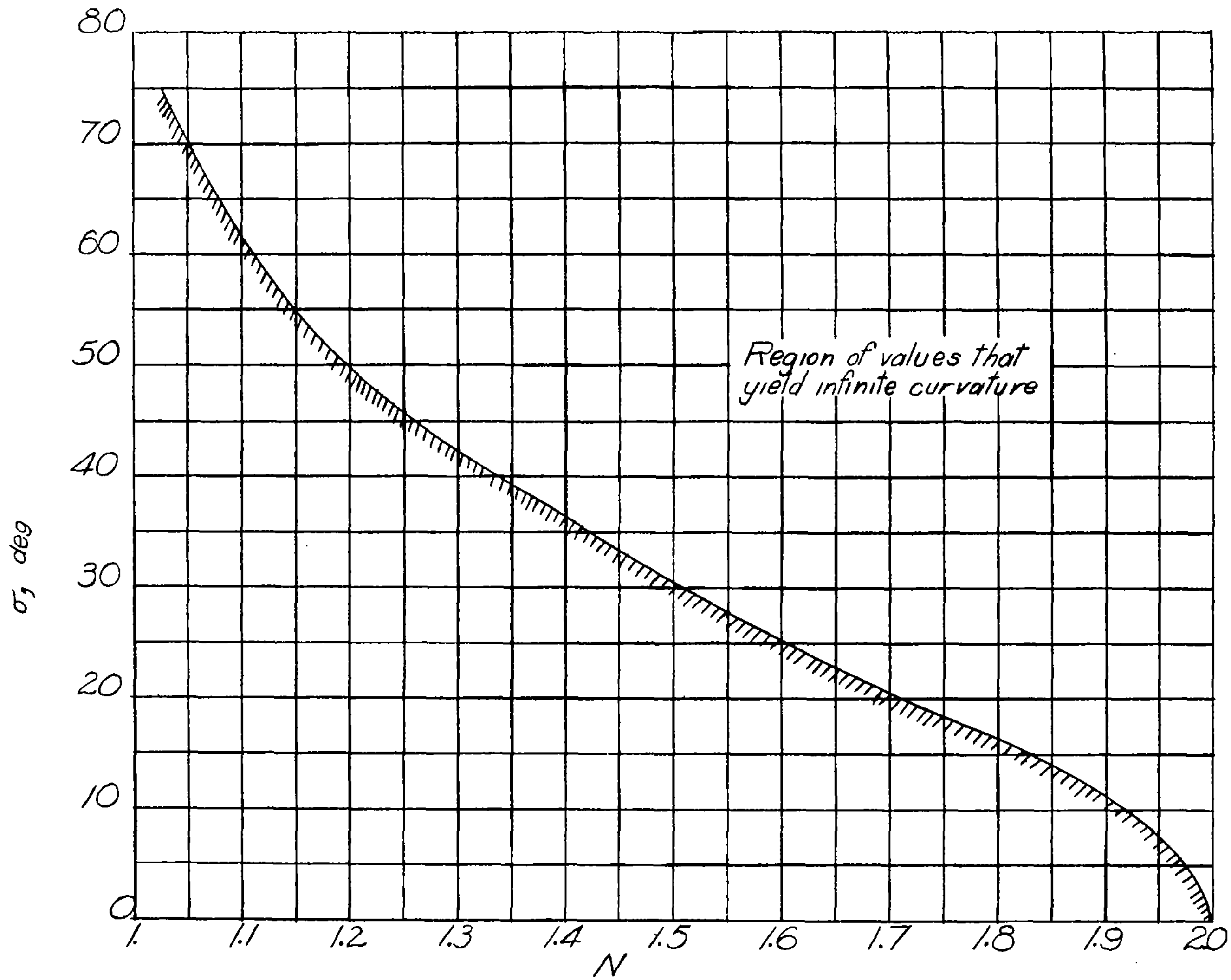


Figure 6.— Corresponding values of N and σ that yield infinite rates of curvature. Constant navigation.

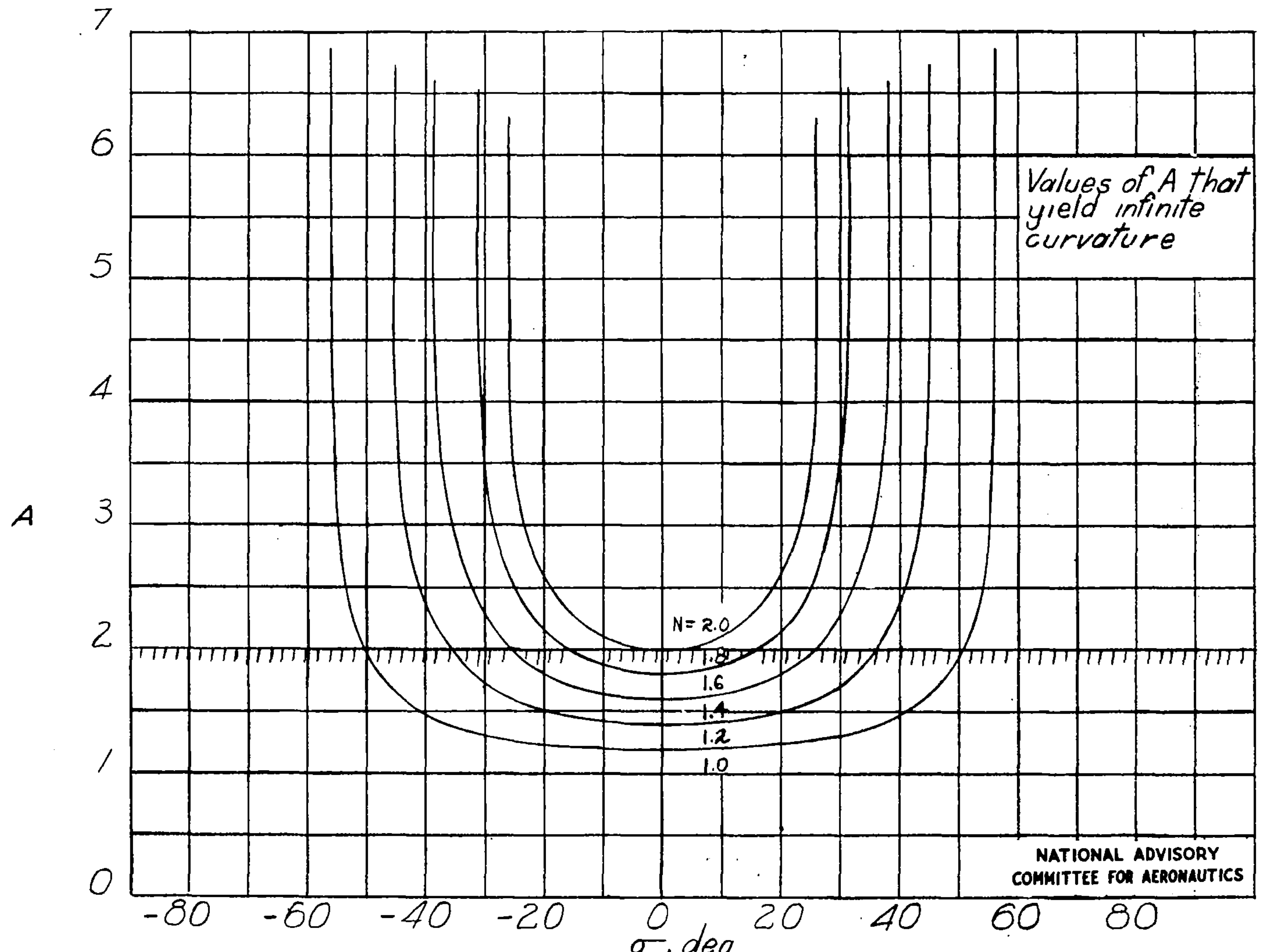
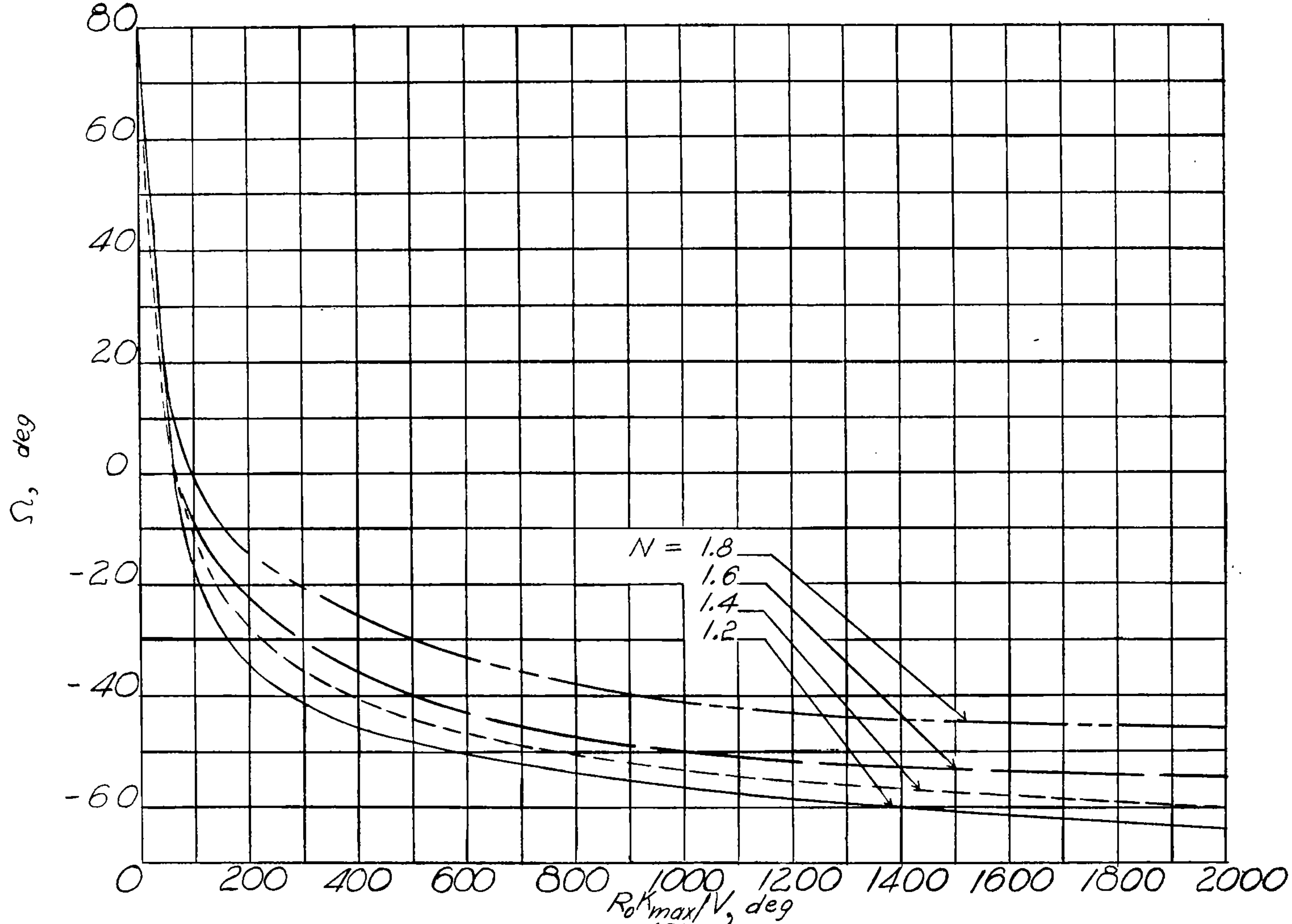


Figure 7.– Variation of A with σ for some particular values of N. Constant navigation.



(a) $\sigma = 10°$.

Figure 8.– Variation of R_0K_{max}/V with Ω for particular values of N and σ. Constant navigation.

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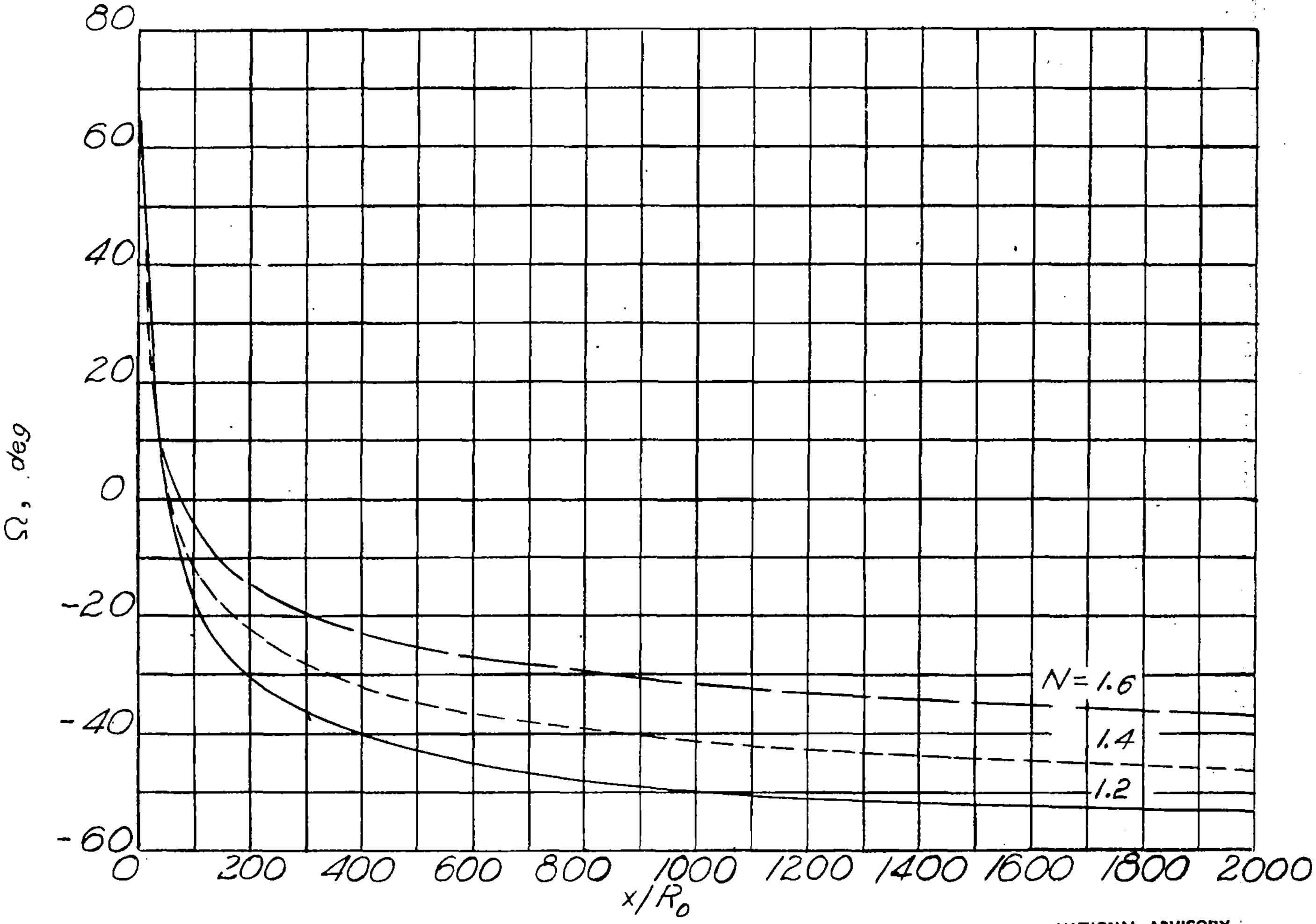
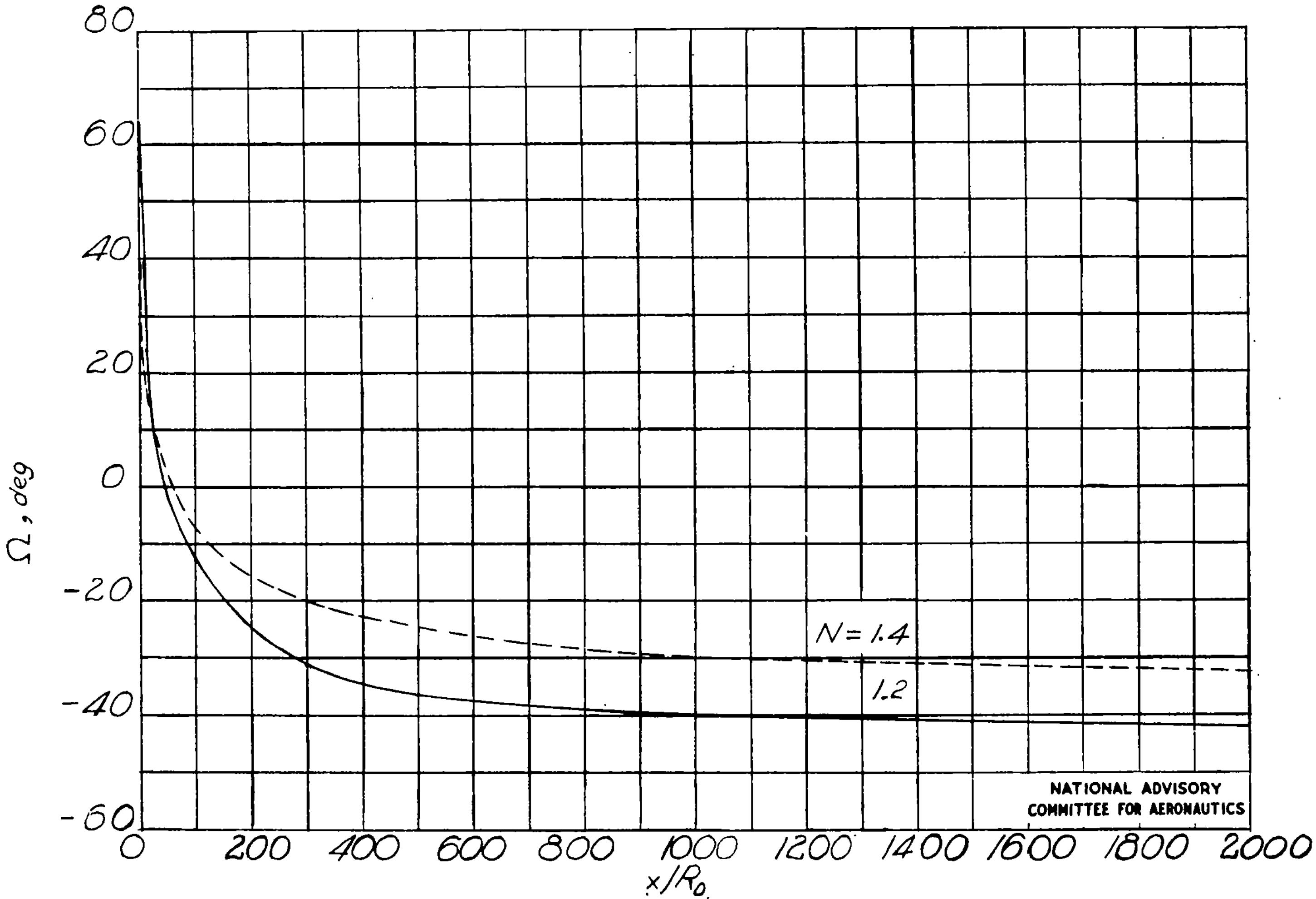
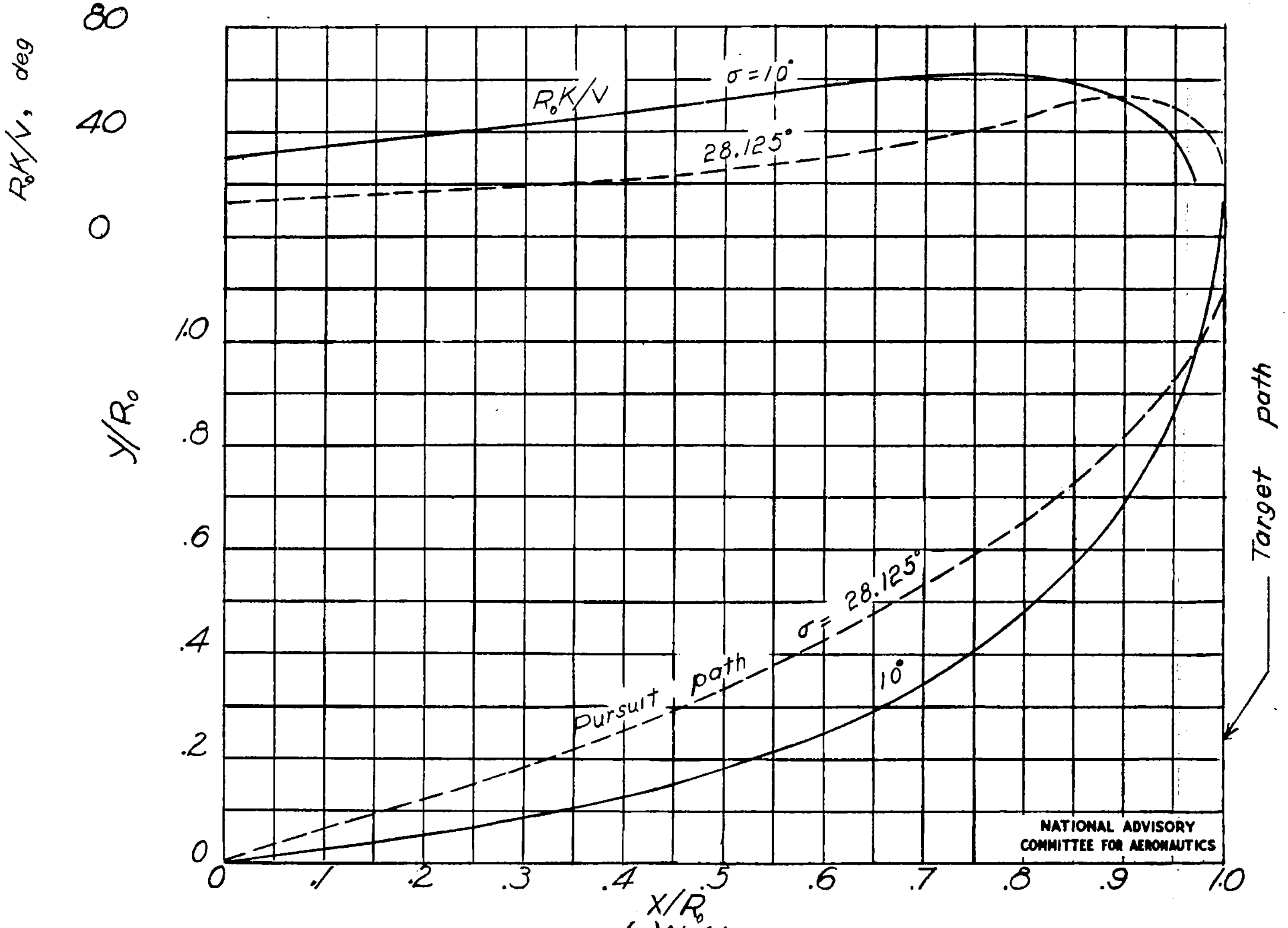


(b) $\sigma = 20°$.

Figure 8.- Continued.

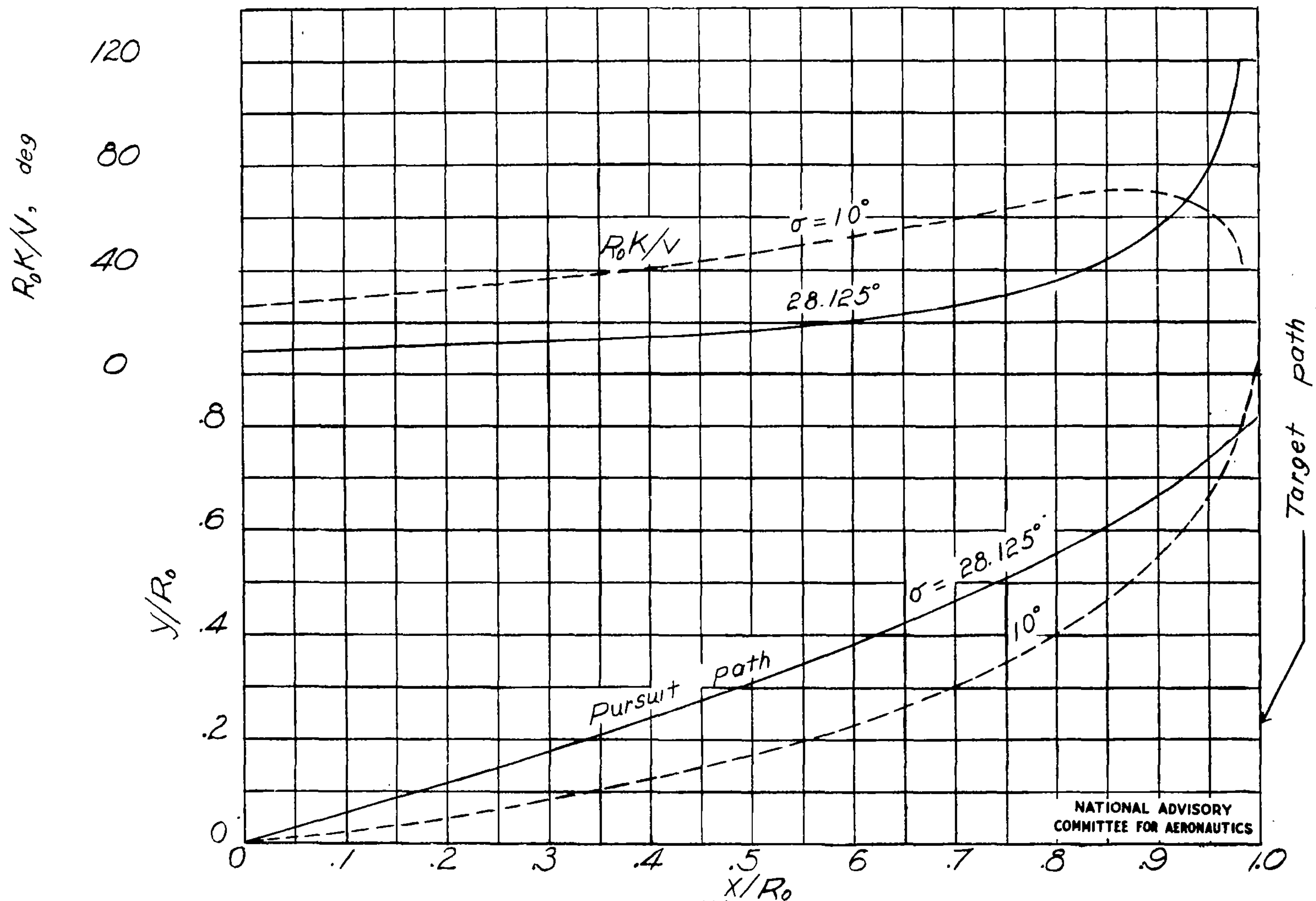


(c) $\sigma = 30°$.
Figure 8.-Concluded.



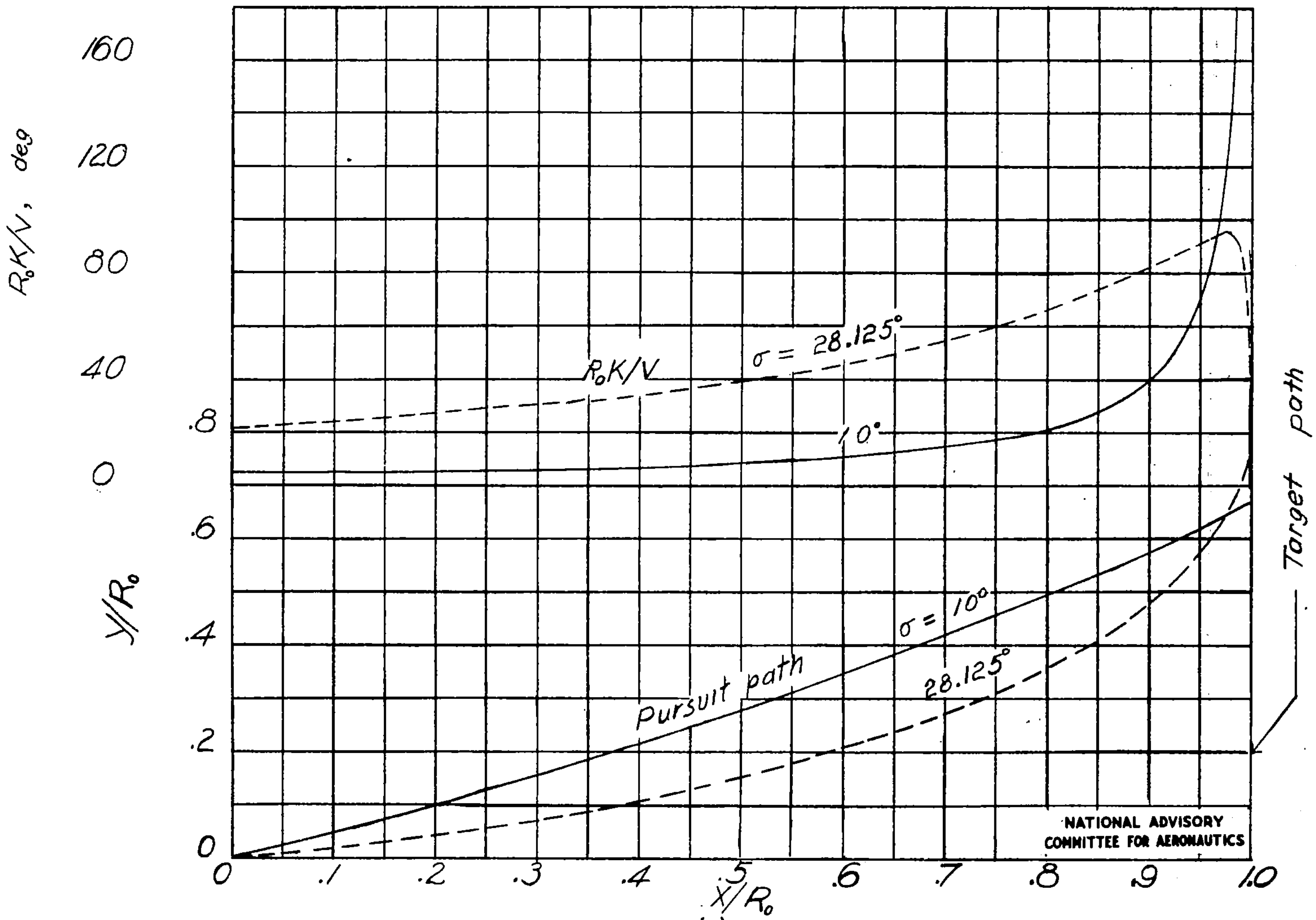
(a) $N = 1.4$.

Figure 9.- Comparison of pursuit paths and corresponding rates of curvature obtained by constant navigation with $\Omega = 0$, $\sigma = 10^\circ$, and 28.125° for different values of N.



(b) $N=1.6$.

Figure 9.- Continued.



(c) N=1.8.

Figure 9.- Concluded.

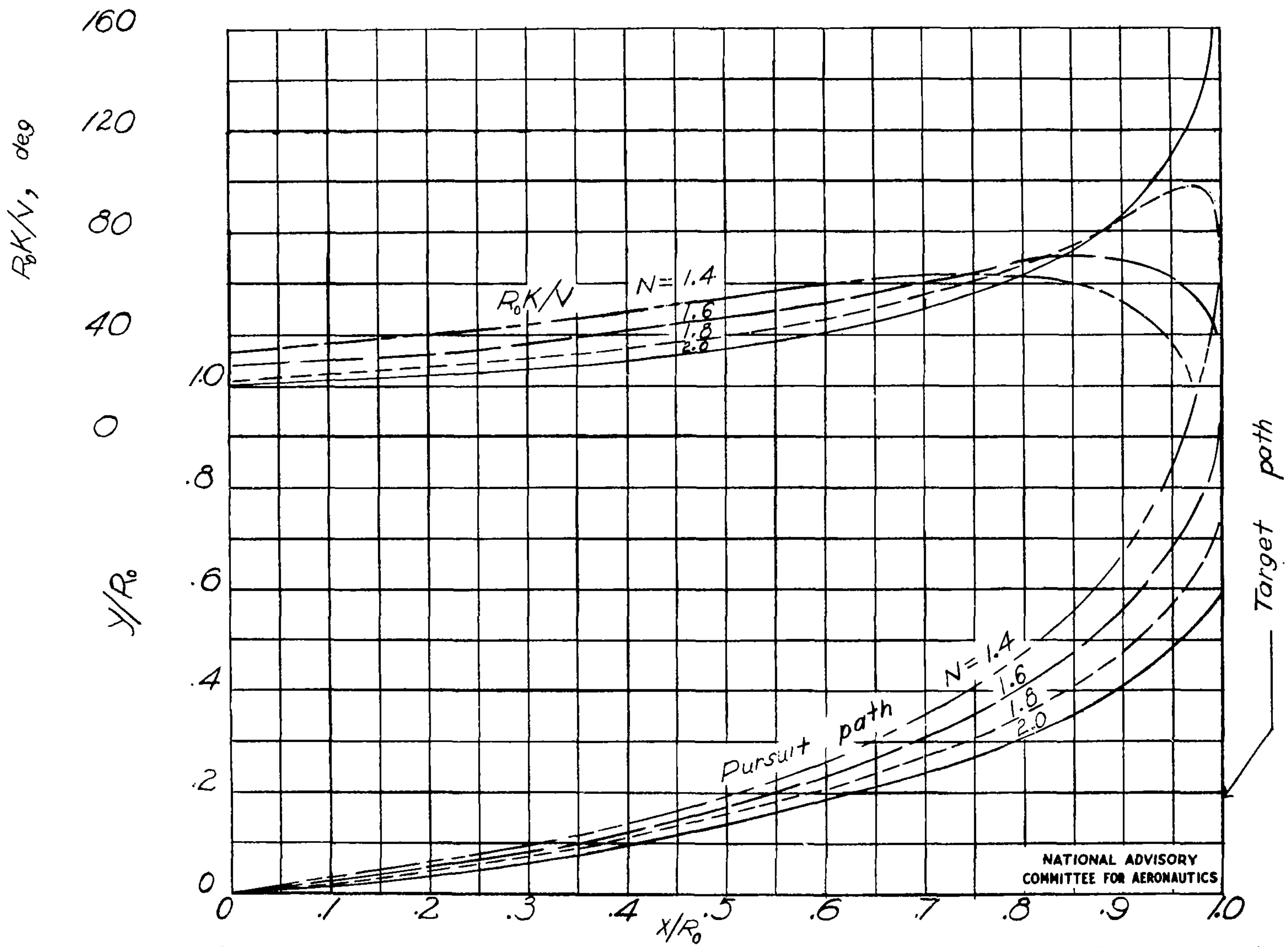
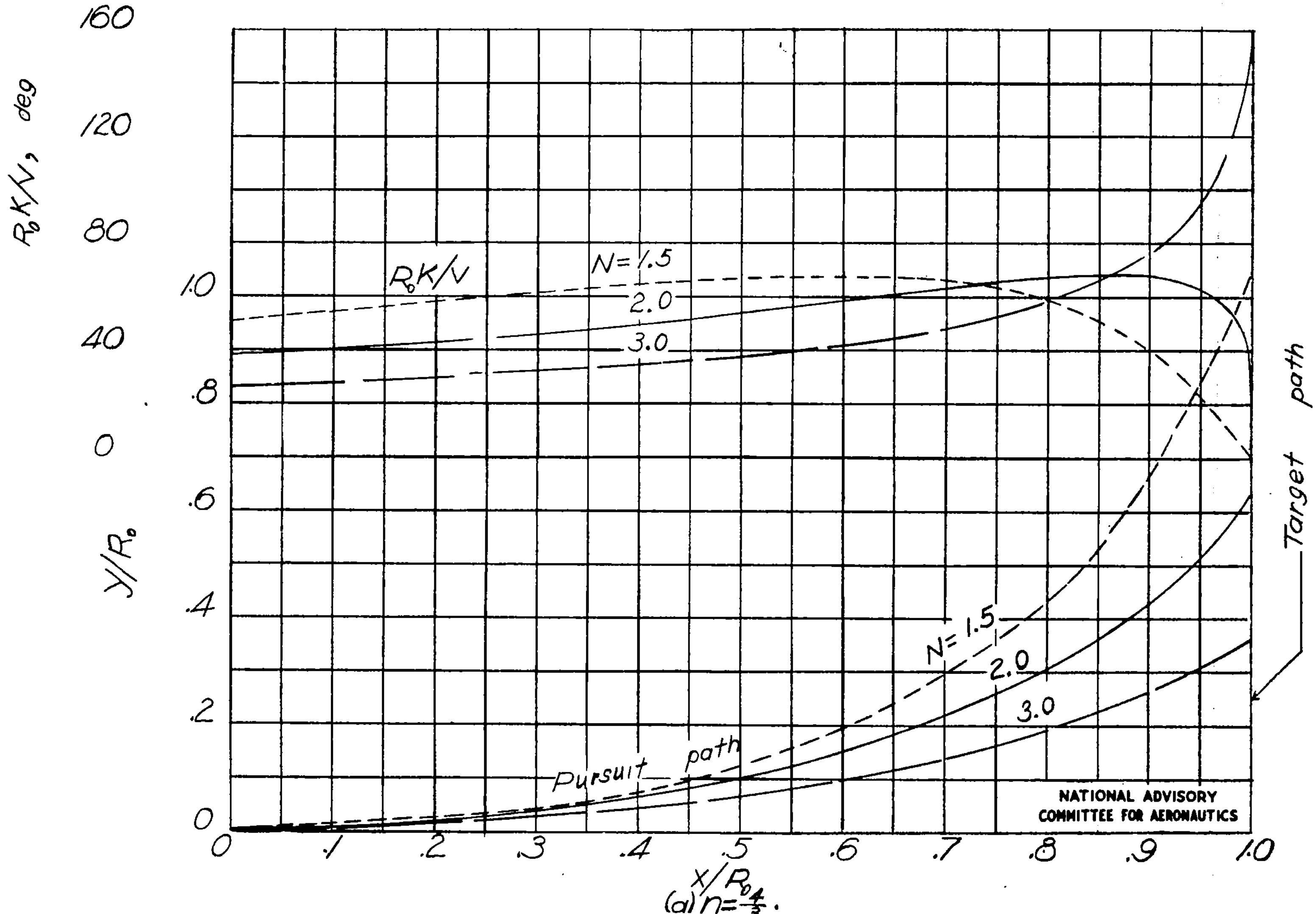


Figure 10. - Comparison of pursuit paths and corresponding rates of curvature obtained by constant navigation with $\Omega = 0$, $\sigma = 10^{\circ}$, for different values of N.



(a) $n = \frac{4}{3}$.

Figure 11.- Comparison of pursuit paths and corresponding rates of curvature obtained by proportional navigation for $\Omega = 0°$ and particular values of n and N.